THERMAL ENGINEERING II

SRI POLYTECHNIC , KOMAND

Prepared by Er. Ranjan K Sahoo

17/9/2016

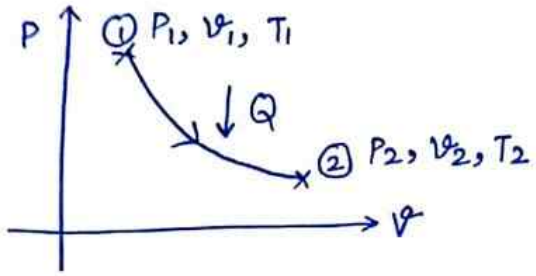
Heat and Mass Transfer

Heat → H.M.T

Heat → T.D.

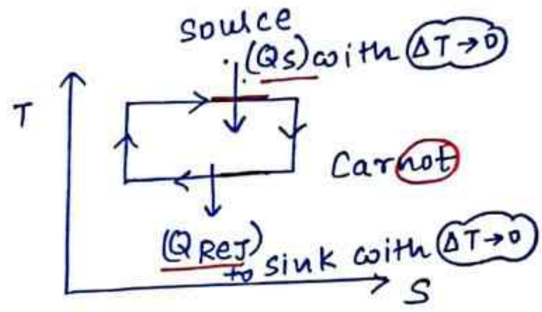
T.D. → Q in Joule

H.M.T → $\frac{\text{Q in Joule}}{\text{sec}}$ = watt → Rate



Rate → H.M.T.

Not T.D → only concern about final and initial tempr or amount of heat.



The main difference b/w Thermodynamic analysis and heat transfer analysis of a problem is that in thermodynamics, we deal with systems in equilibrium that is to bring a system from one equilibrium state to another equilibrium state, how much heat is required is the main criteria in thermodynamic analysis. But in heat transfer analysis we evaluate at what rate this change of state occurs by calculating rate of heat transfer in Joule/sec or watt.

MODES OF HEAT TRANSFER :-

① Conduction.

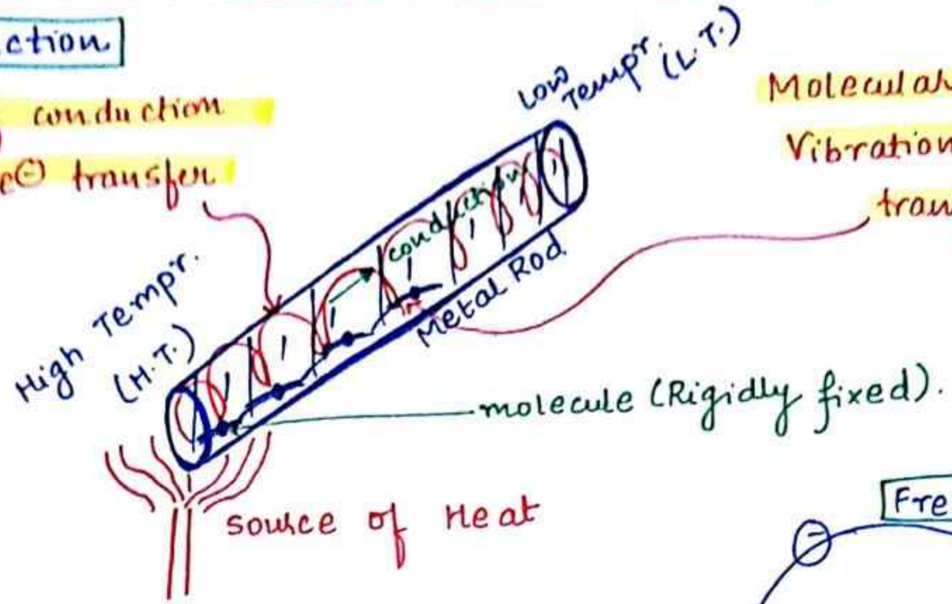
② Convection.

③ Radiation.

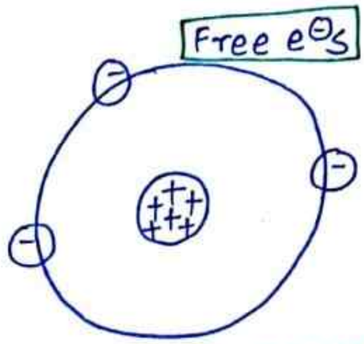
Conduction

(70%) of conduction
free $e^{\ominus}$ transfer

Molecular Lattice
Vibrational energy
transfer (30%)



30% and 70% only for metallic Bodies.



free $e^{\ominus}$s take energy & goes to excited state.

Best known metallic conductor :-

Silver → $K = 410 \frac{W}{m-K}$

then

Copper → $K = 385$ W/mK

Aluminium → $K = 200$ W/mK
(Low ρ)
density

Steel → $K = 17$ to 45 W/mK

Have plenty of free $e^{\ominus}$s.

All are metals &

also,

$$K_{pure\ metal} > K_{its\ alloy}$$

Ex :-

$$K_{Iron} > K_{steel}$$

$K_{copper} > K_{Brass}$ and Bronze also.

Conduction :- is the mode of heat transfer which generally occurs in a solid body due to tempr. difference associated with Molecular Lattice vibrational energy transfer and also by free $e^{\ominus}$ transfer.

The reason behind all electrically good conductors are also in general good conductors of heat is that the presence of abundant (plenty / lot of) free electrons. $\boxed{Ex}$:- All Metals ③

$\boxed{Notable}$ Exception to the above statement is :- diamond whose thermal conductivity is 2300 W/mK

non-metal

DIAMOND → Due to molecular lattice arrangement

↳ order of arrangement perfect.

The highest thermal conductivity of diamond is due to its perfect crystalline molecular lattice arrangement.

NON-METALS

crystalline

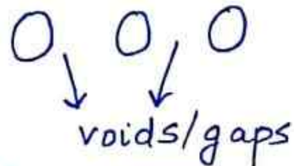
orderly, Intimate arrangement of molecules.

$\boxed{Ex}$:- ① Diamond

② quartz

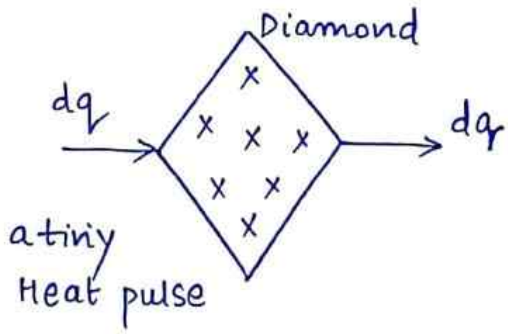
③ Graphite

Amorphous



$\boxed{Ex}$:- Glass

$K_{glass} = 1.5$ W/mK



$\boxed{\text{THERMAL CONDUCTIVITY DEF}^{N.} (K)}$:- K is a thermophysical property of a material which tells about the ability of the material to allow the heat energy to get conducted through the material more rapidly or quickly.

(Thermophysical property)

Insulators have very low thermal conductivity thereby prevent the conduction heat transfer rate.

Scanned by CamScanner

Ex - (1) Asbestos → K = 0.2 W/mK

(2) Refractory Brick → K = 0.9 W/mK
(Furnaces)

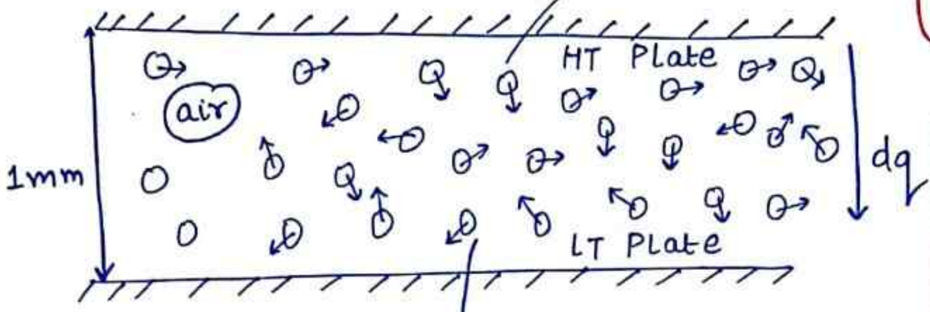
(3) Glass wool → K = 0.075 W/mK

(4) Polyurethene foam → K = 0.02 W/mK
(PUF)
(Refrigerator walls)

HEAT CONDUCTION IN GASES :-

High Tempr., high velocity Molecules

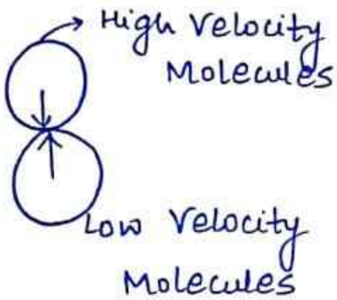
HT → high Tempr.
LT → low Tempr.



From kinetic theory of gases
$\bar{c} \propto \sqrt{T}$
(RMS velocity) of gas molecule

low Tempr., low velocity molecules

Molecular Momentum transfer during elastic collision.



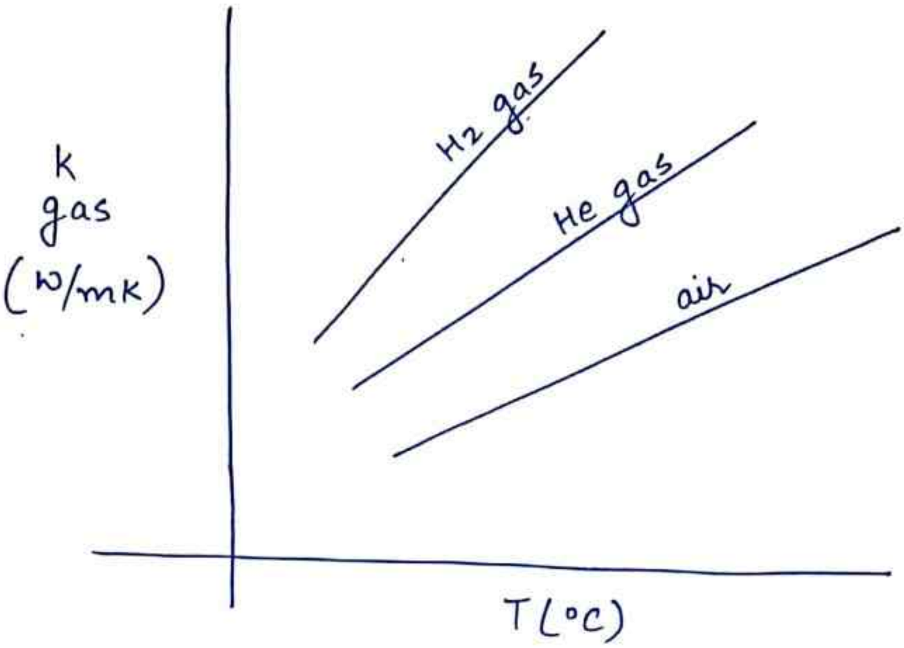
air is stagnant
gap is very narrow
→ rest

Heat conduction occurs in gases by molecular momentum transfer when high velocity, high tempr. molecules collide with the low velocity low tempr. molecules but in general gases are very bad conductors of heat.

$$\boxed{K_{air} = 0.026\ W/mK \quad \text{(at Room conditions)}}$$

As the tempr. of the gases increase, their thermal conductiv. also increased because at higher temperatures of gas, ~~rate of~~ greater molecular activity may result in more no. of collisions per unit time and hence more momentum transfer rate.

5



As $T_{gas} \uparrow$

$\Rightarrow K_{gas} \uparrow$

(m^2/sec) ν (K.V.) $\uparrow$ → Kinematic viscosity

$C_p \uparrow$

$\rho \downarrow$

Liquids are better conductors of heat than gases.

$$K_{water} = 0.63\ W/mK$$

Among all the liquids,

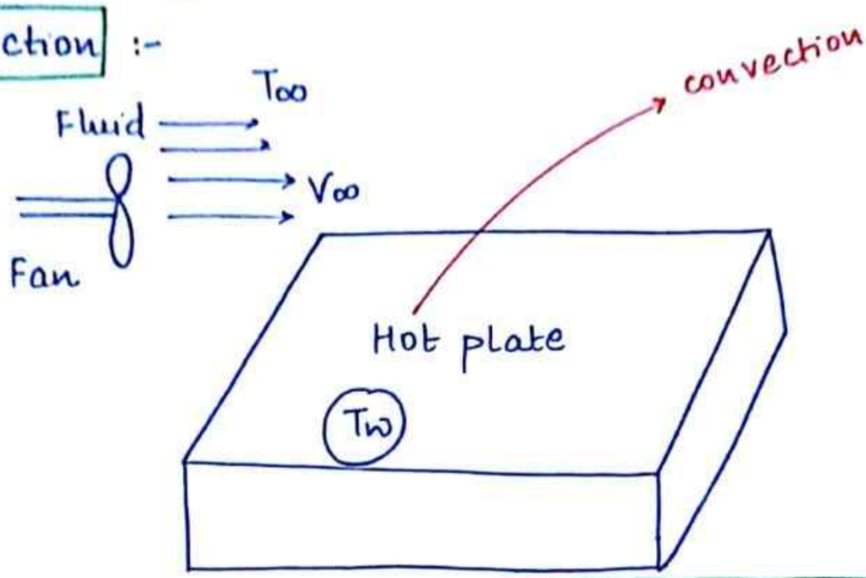
Mercury (Liquid metal) has highest thermal conductivity

$$K_{Hg} = 8.43\ W/mK$$

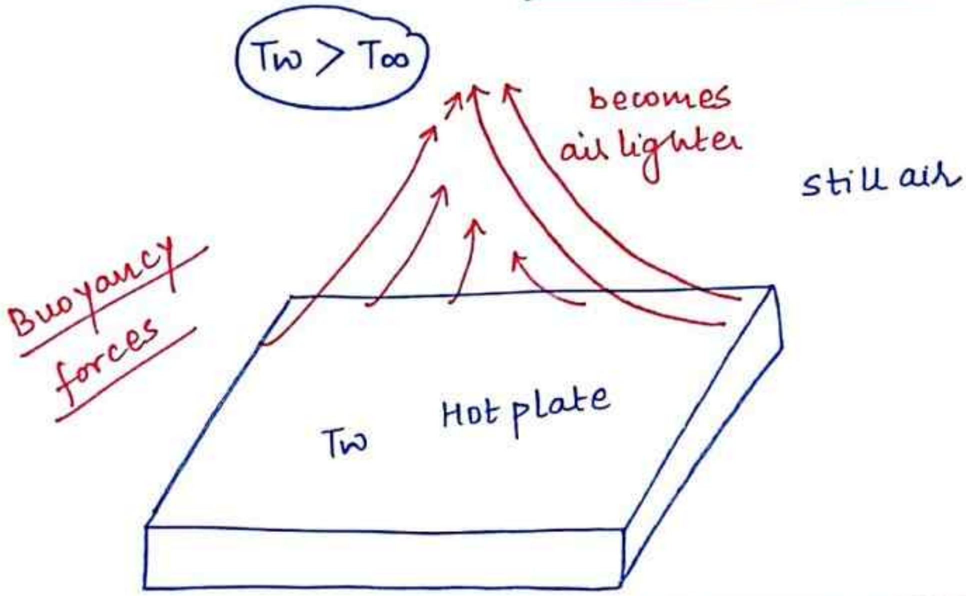
Hg (Thermometric fluid) → due to good conduction, good expansion, good low vapour pressure.

Hg → low V.P.
→ good volume expansion with heating
→ high 'k'.

$\boxed{\text{Convection}}$:-



Forced convection



Free/Natural convection Heat Transfer (H.T.)

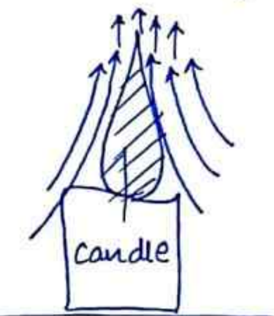
$$\frac{P}{\downarrow \rho} = RT\uparrow$$

Fluid can transfer/transport thermal energy in the form of enthalpy manifested by its Temp. ($\dot{m} c_p T$)

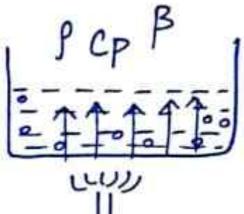
Convection is the mode of heat transfer which generally occurs between a solid surface and the surrounding fluid due to temperature difference associated with macroscopic Bulk motion of the fluid transporting thermal Energy. In case of forced convection heat transfer, this motion of the fluid is provided by an external agency like a fan or a blower or a pump whereas in free convection heat transfer, the motion of the fluid occurs naturally due to Buoyancy forces arising out of density changes of fluid (because of its tempr. change).

Conduction → internal phenomenon.

Convection → Boundary —— " ——.



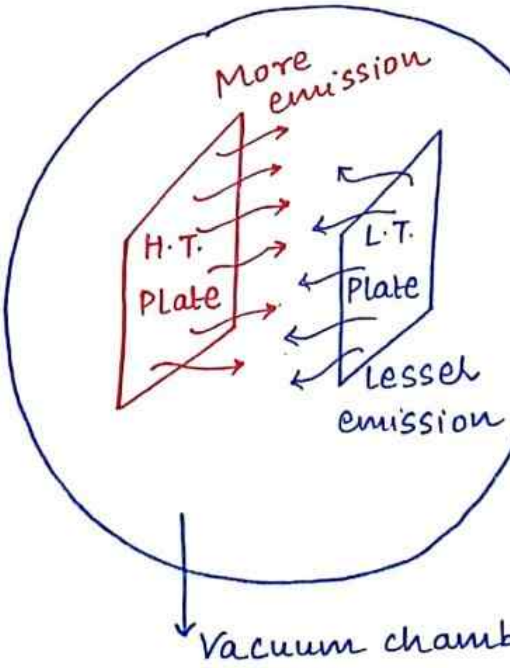
FREE CONVECTION



FORCED

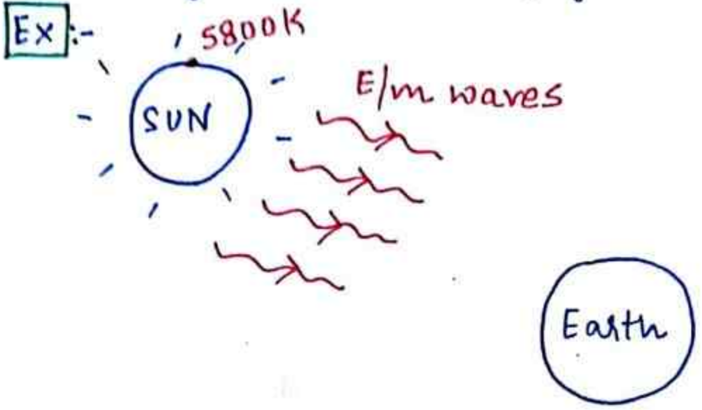
RADIATION :→

- All bodies at all Temperatures emit Thermal Radiation except the body at 0K (−273.15°C). (∵ $\eta \neq 100\%$ − carnot) (violates IInd Law)
- The rate of emission occurs in the form of electromagnetic waves which can propogate even through vacuum.
- The rate of energy emission $\propto T^4$ (T in K).

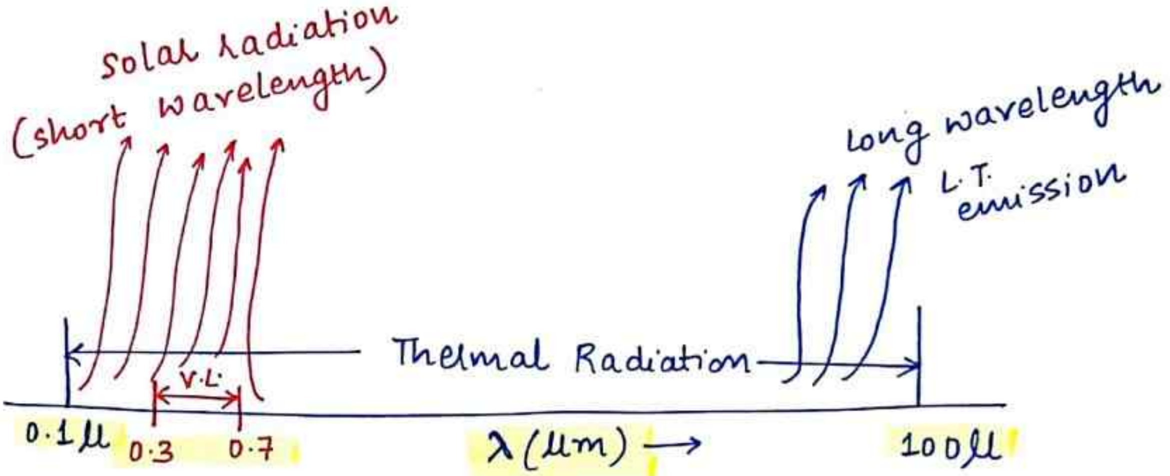


Radiation is the mode of heat transfer which does not require any material medium for its propogation and hence occurs by electromagnetic wave propogation travelling with a speed of light.

Ex:-



V.L. → visible light



E/m spectrum

$$T_K \longrightarrow E \xrightarrow{watt/m^2} \lambda(\mu m)$$

T_K ↓ high

E ↓ very high

λ ↓ small

NOTE :-

Radiation Mode of heat transfer completely predominates over conduction & convection particularly when the tempr. difference is sufficiently large.

Ex :- The Mode of H.T. between hot flue gases and Refractory brick walls in a large pulverised fuel fired Power Boiler is predominantly by Thermal Radiation.

Reason :- Large ΔT.

$$q_{cond} \propto (T_1 - T_2)\ ^\circ C \text{ (OR) } K$$

$$q_{conv} \propto (T_w - T_\infty)\ ^\circ C \text{ (OR) } K$$

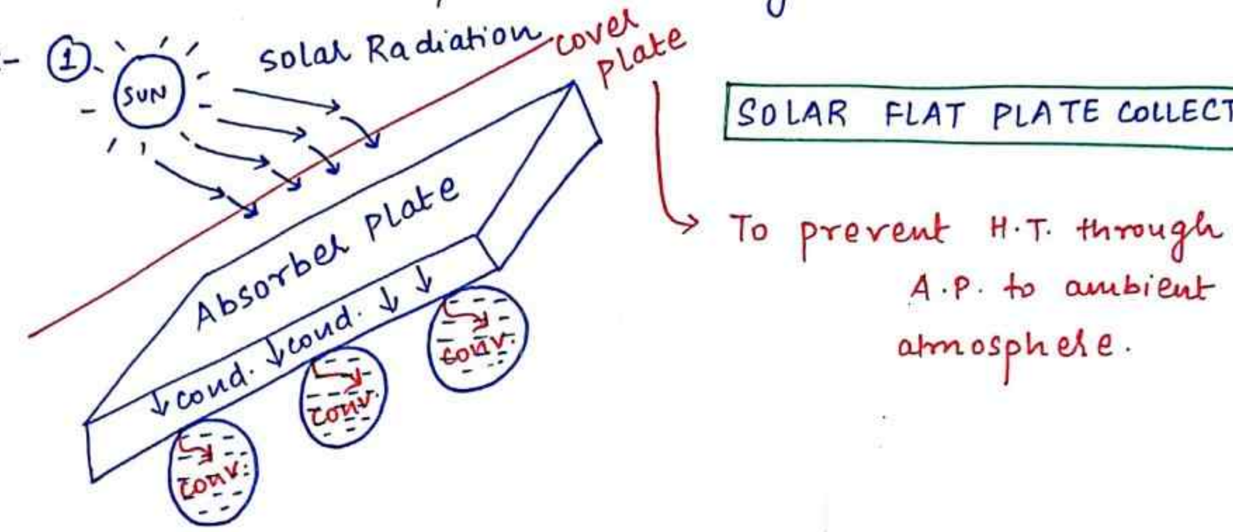
$$q_{Radiation} \propto (T_1^4 - T_2^4)\,\sigma$$

σ → Steffan's Boltzman constant

T_1, T_2 → Kelvin

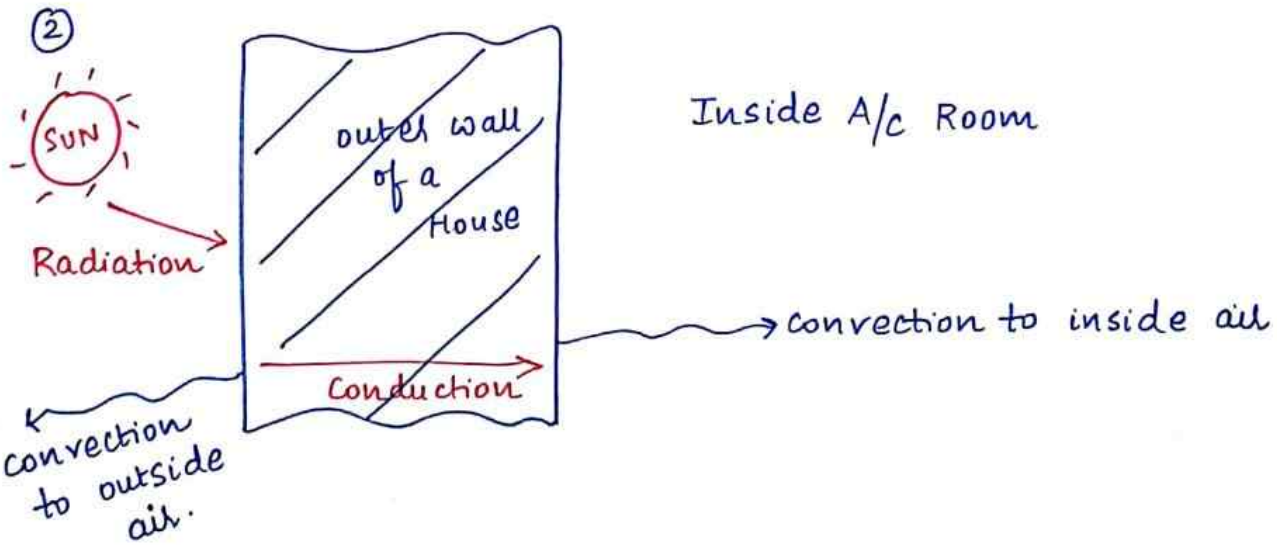
NOTE :- In any practical situation of heat transfer all the 3 modes of heat transfer may simultaneously exist.

Ex :- ①

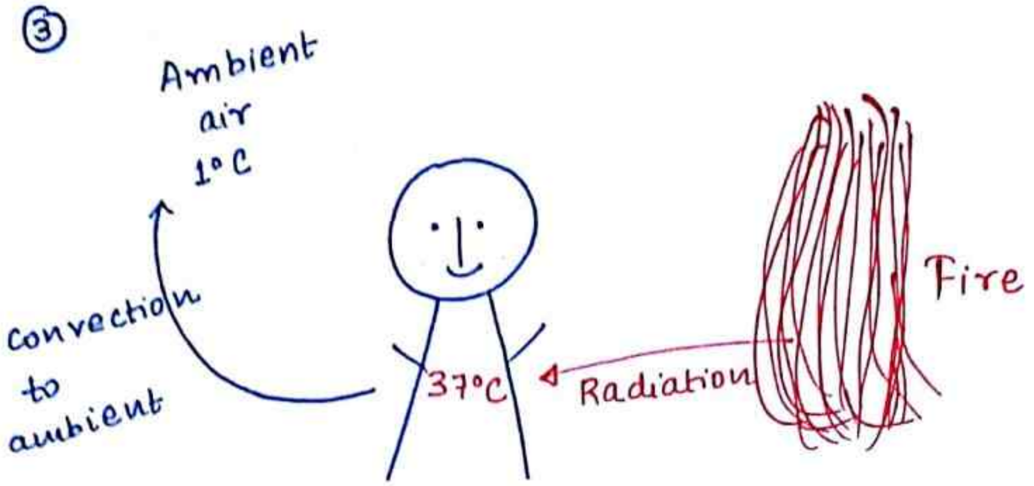


SOLAR FLAT PLATE COLLECTOR

②

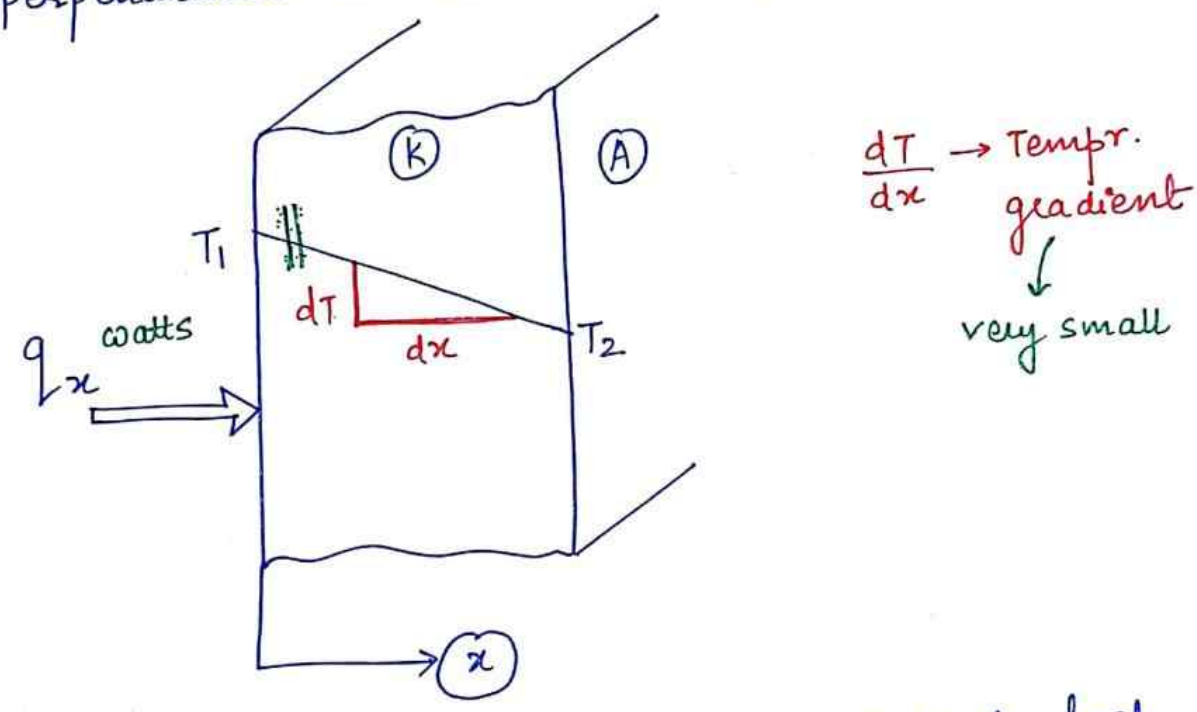


③



* $\boxed{\text{GOVERNING LAWS OF HEAT TRANSFER}}$:-

① $\boxed{\text{Fourier's law of Conduction}}$:- The law states that the rate of heat transfer by conduction along a given direction is directly proportional to the tempr. gradient along that direction and is also directly proportional to the area of heat transfer lying perpendicular to the direction of heat transfer.



$$q_x \propto -\frac{dT}{dx}$$

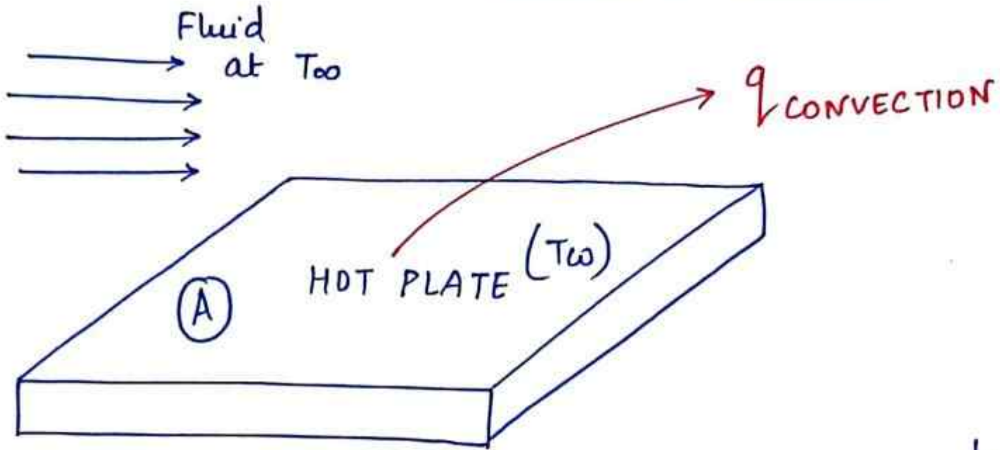
(-ve sign shows that heat always flows in the direction of decreasing Temp. i.e. to satisfy clausius's statement of IInd law of T.D.)

$\propto A$

Rate of heat transfer (H.T.) $\boxed{q_x = -KA\left(\frac{dT}{dx}\right) \text{ watt}}$

$K \rightarrow$ Thermophysical property of Material of slab.

② NEWTONS LAW OF COOLING (for CONVECTION H.T.) -



The law states that the rate of heat transfer by convection between a solid body and a surrounding fluid is directly proportional to the tempr. difference between them and is also directly proportional to the area of contact or area of exposure between them.

$$q_{conv.} \propto (T_\omega - T_\infty)$$

$$\propto A$$

$$\boxed{q_{conv.} = hA(T_\omega - T_\infty) \text{ watt}}$$

h = convection heat transfer coefficient

(OR)

Film H.T. coefficient $\left(\text{watt}/m^2K\right)$. SI

also $= \text{watt}/m^2\,{}^\circ C$.

Scanned by CamScanner

NOTE :- unlike thermal conductivity K, h is not a property of the material but it depends upon some of the thermophysical properties of the fluid like ρ, μ, c_p, K

In forced convection, H.T.

$$\boxed{h = f(\vec{V}, D, \underbrace{\rho, \mu, c_p, k}_{})}$$

Thermophysical properties of fluid. → which can influence our H.T. phenomenon.

$\vec{V}$ = Velocity of fluid.

D = characteristic Dimension of Body.

$\rho_w = 1000 \rho_a$

$C_{p_w} = 4.186\ KJ/Kg\,K$

$\mu_w > \mu_{air}$

$K_w > K_{air}$

} water v/s air

In free convection H.T.,

$$\boxed{h = f(g, \beta, \Delta T, L, \underbrace{\mu, \rho, c_p, k}_{\text{properties of fluid}})}$$

g → Acc^n due to gravity.

β → Isobaric volume expansion coefficient of fluid.

$\Delta T = T_w - T_\infty$

L = characteristic Dimension of Body.

* Ranges of 'h' :-

① Free Convection in Gases :- h = 3 to 25 Watt/m^2K.

② Forced Convection in gases :- h = 25 to 400 W/m^2K.

③ ~~Forc~~ Free Convection in liquids :- h = 250 to 600 W/m^2K.

④ Forced convection in liquids :- h = 600 to 4000 W/m^2K

⑤ Condensation Heat transfer (vap. to liquid) :- h = 3000 to 25,000 W/m^2K

⑥ Boiling Heat Transfer (Liq. to vapour) :- h = 5,000 to 50,000 W/m^2K

* ③ STEFAN - BOLTZMAN's Law of Radiation :-

The law states that the radiation energy emitted from the surface of a Black Body per unit time per unit area is directly proportional to the 4th power of the absolute tempr. of the Black Body.

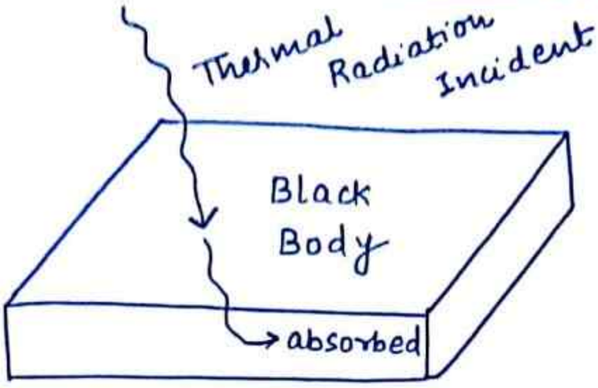
$$E_b \propto T^4 \text{ (T in kelvin only)}$$

$$\boxed{E_b = \sigma T^4 \ \frac{\text{Joule}}{\text{sec } m^2} = \frac{\text{watt}}{m^2}}$$

σ = stefan - Boltzman's constant

$\sigma = 5.67 \times 10^{-8}$ Watt/m^2K^4

Black body is the Body which absorbs all the thermal Radiation incident or falling ~~above~~ or upon the body

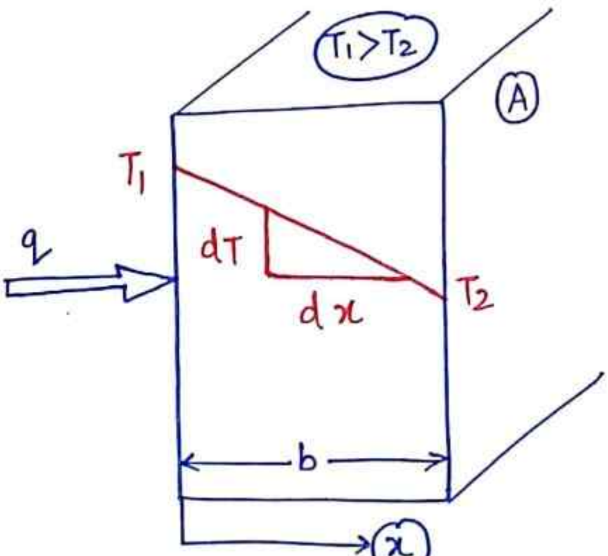


Black Body → Perfect absorber.
→ Ideal- emitter.
→ Diffusive in nature.

NOTE :- A Thermally black body absorbing all the incident thermal radiation falling upon it may not appear black in colour to the human eye.

Ex:- Ice and Snow.

CONDUCTION H.T. (Integration of Fourier's Law of Convection)



Assuming :- ① steady state H.T. conditions

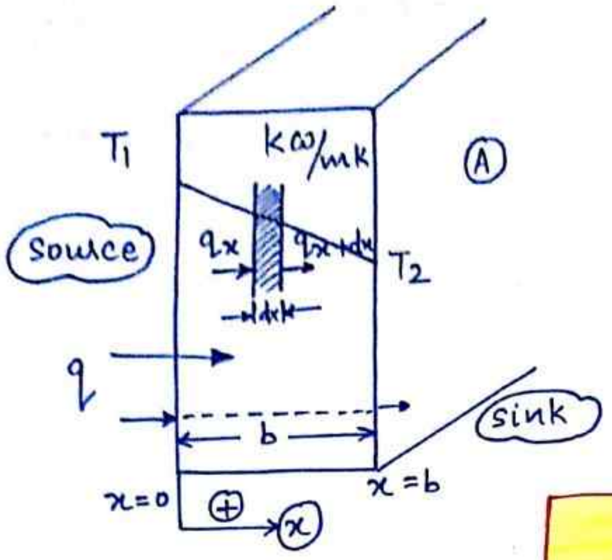
$T \neq f(\text{Time})$

② one - dimensional heat conduction

$T = f(x)$

at $x = 0 \Rightarrow T = T_1$
at $x = t \Rightarrow T = T_2$
} Boundary cond^n's of a slab

③ uniform (or) constant 'K' of material.



$$\int_{x=0}^{x=b} q_x \, dx = \int_{T_1}^{T_2} -KA \, dT$$

$q_x \neq f(x)$ to satisfy steady state conditions.

i.e. $q_x = q_{x+dx}$

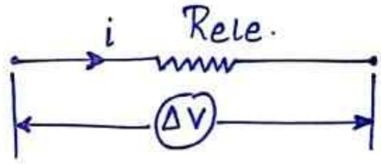
$\Rightarrow q_x \times b = KA(T_1 - T_2)$

$\Rightarrow$ Rate of conduction H.T. through slab $= q_x = \dfrac{KA(T_1 - T_2)}{b}$ watt

$\Rightarrow$ q/A = H.T. Rate per unit area (or) Heat flux $= \dfrac{K(T_1 - T_2)}{b}$ watt/m^2

* Electrical Analogy of Heat Transfer :-

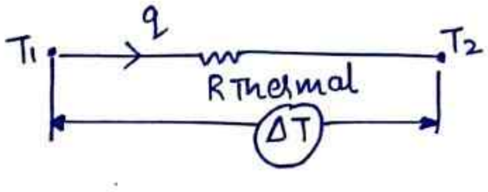
Electrical	Thermal
i (Amp) $\longrightarrow$	q (watt)
(ΔV) (or) Emf (volts) $\longrightarrow$	ΔT (°C) or K
R_{elec} (Ω) $\longrightarrow$	$R_{Thermal}$



ohm's law :-

$$R_{elec} = \frac{\Delta V}{i} \, \Omega$$

Thermal circuit



$$R_{Thermal} = \left(\frac{\Delta T}{q}\right) \text{K/watt}$$

$$\therefore \underset{(\text{slab})}{(R_{Th})_{\text{conduction}}} = \frac{T_1 - T_2}{(q)} = \frac{(b)}{(KA)}\ K/\text{watt}$$

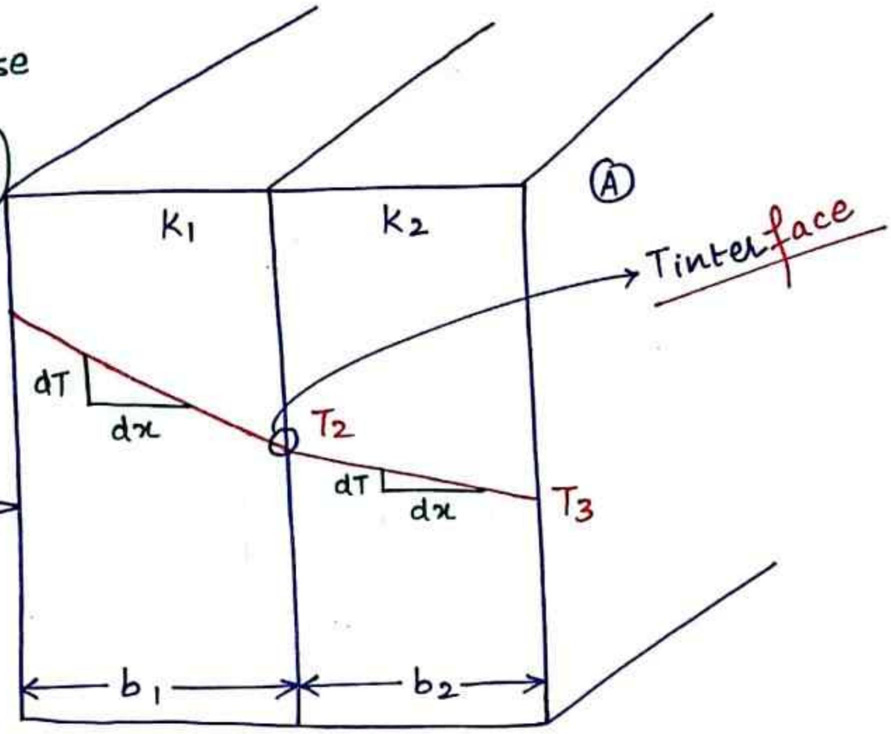
$\boxed{\text{NOTE}}$:- If the thickness of the slab is more and if its conductivity is low, then the conduction thermal resistance offered by the slab will be higher, hence heat current will be lesser.

* $\boxed{\text{Conduction heat transfer through a composite slab}}$:-

$K_1 < K_2$ because $\left(\frac{dT}{dx}\right)_{\text{in }①} > \left(\frac{dT}{dx}\right)_{\text{in }②}$

(q being same throughout)

$q = -KA\frac{dT}{dx}$



assume steady state one dimensional conduction H.T through the composite slab.

$\boxed{\text{Thermal circuit}}$:-

$$T_1 \xrightarrow[(b_1/K_1A)]{q \text{ watt}} T_2 \xrightarrow[(b_2/K_2A)]{} T_3$$

$$q = \frac{T_1 - T_2}{(b_1/KA)} \Rightarrow T_2 = \boxed{}\ ^\circ C$$

(or) $$q = \frac{T_2 - T_3}{(b_2/K_2A)} \Rightarrow T_2 = \boxed{}\ ^\circ C$$

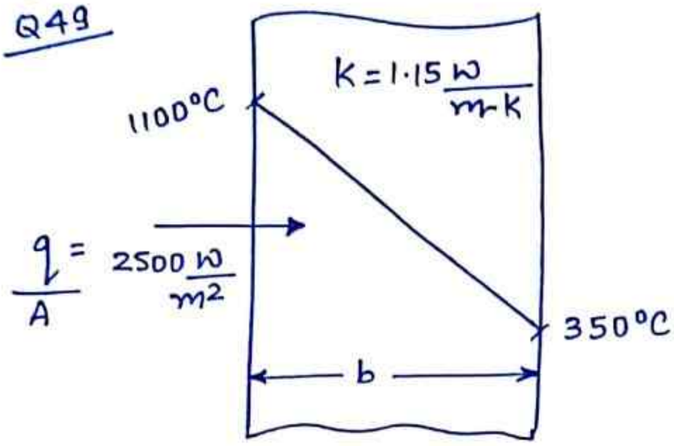
$\boxed{\text{Note}}$:- Area of H.T. will NOT change in the direction of heat flow in case of slabs.

Scanned by CamScanner

$\therefore$ Rate of conduction H.T. through composite slab $= q = \dfrac{(T_1 - T_3)}{\dfrac{b_1}{K_1 A} + \dfrac{b_2}{K_2 A}}$ watt.

$\therefore$ q/A = Heat flux $= \dfrac{T_1 - T_3}{\dfrac{b_1}{K_1} + \dfrac{b_2}{K_2}}$ watt/m^2

WB
Pg 70
Q49

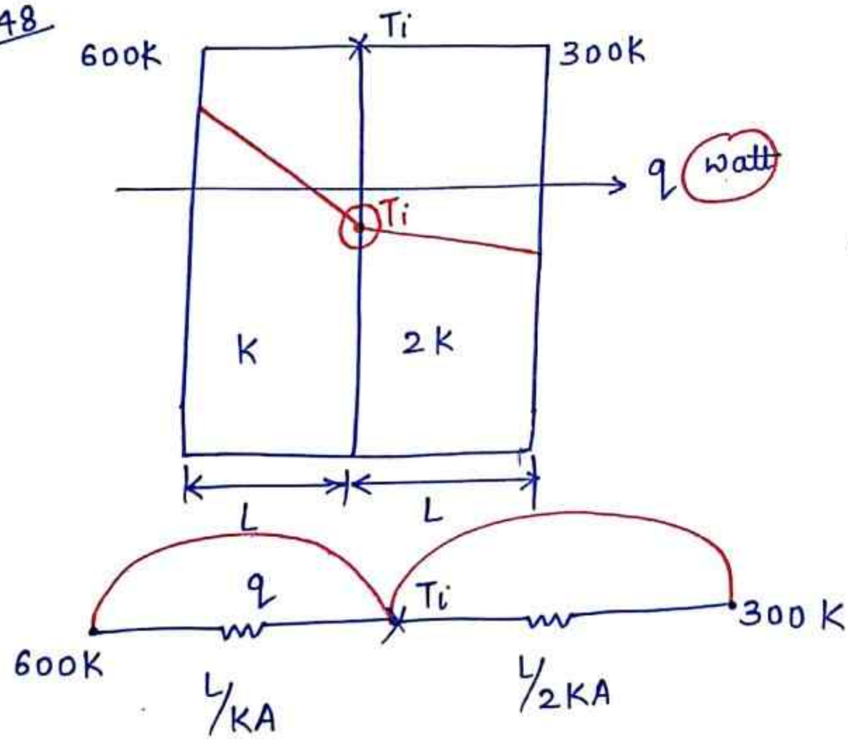


q/A = Heat flux $= \dfrac{K(T_1 - T_2)}{b}$

$\Rightarrow 2500 \dfrac{W}{m^2} = \dfrac{1.15\,(1100 - 350)}{b}$

$\Rightarrow b = 0.345 m$

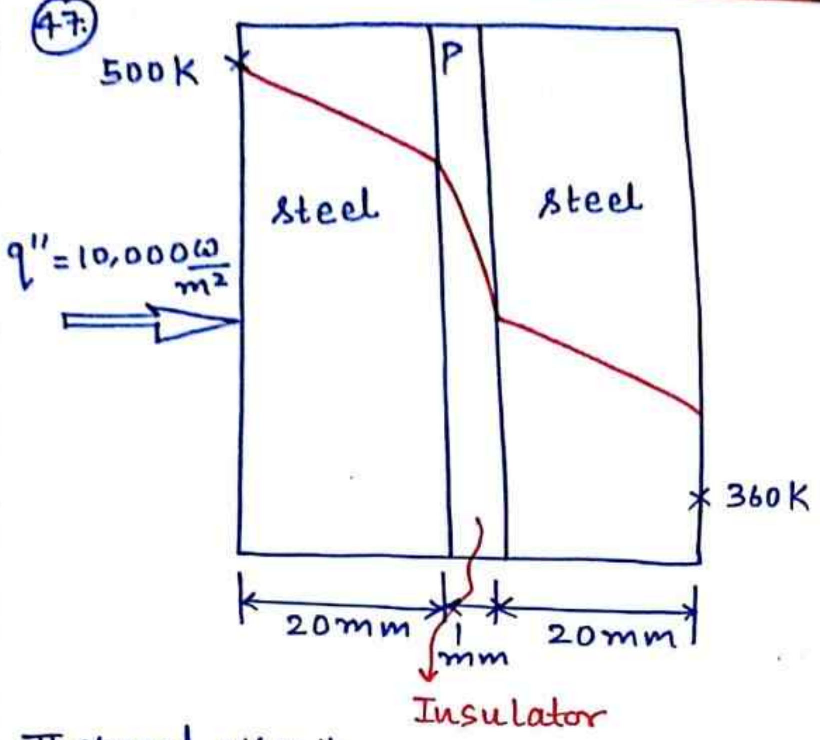
Q48



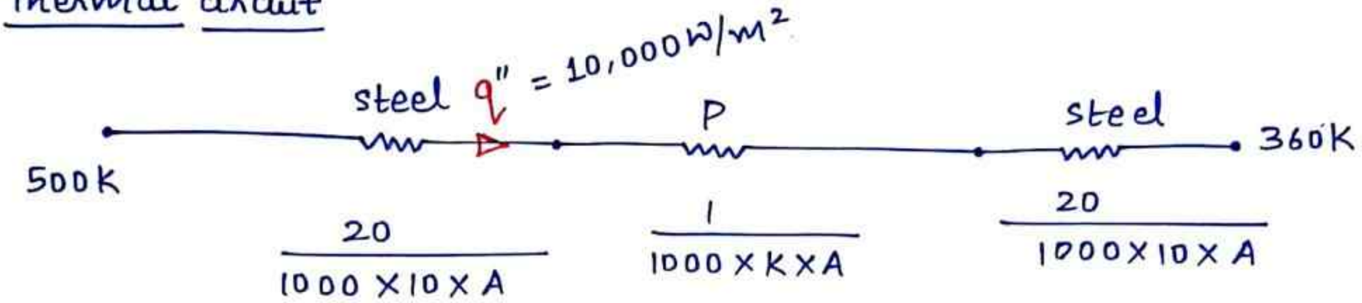
$q = \dfrac{600 - T_i}{\cancel{L/KA}} = \dfrac{T_i - 300}{\cancel{L/2KA}}$

$T_i = 400K$

47.



Thermal circuit

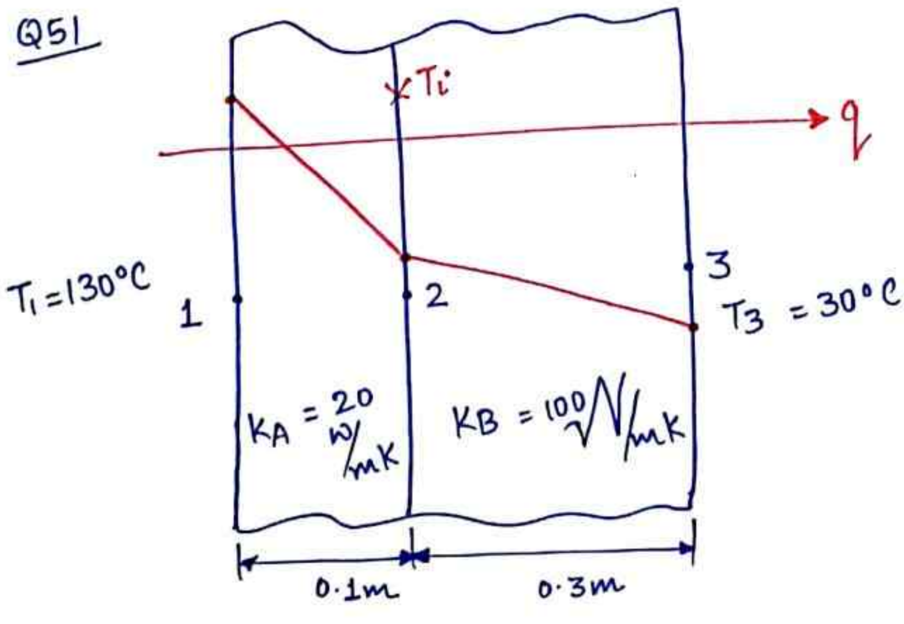


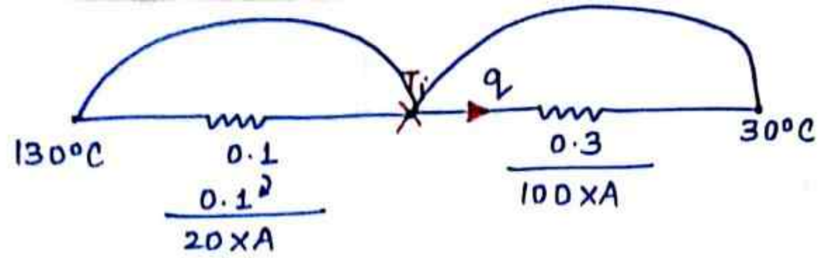
$$\therefore \text{ Heat flux} = q/A = \overset{\text{flux}}{q''} = \frac{500-360}{\frac{200}{10{,}000} + \frac{1}{1000K} + \frac{20}{10{,}000}} \, W/m^2$$

$$= 10{,}000 \, W/m^2$$

$$K = 0.1 \, W/mK$$

Q51

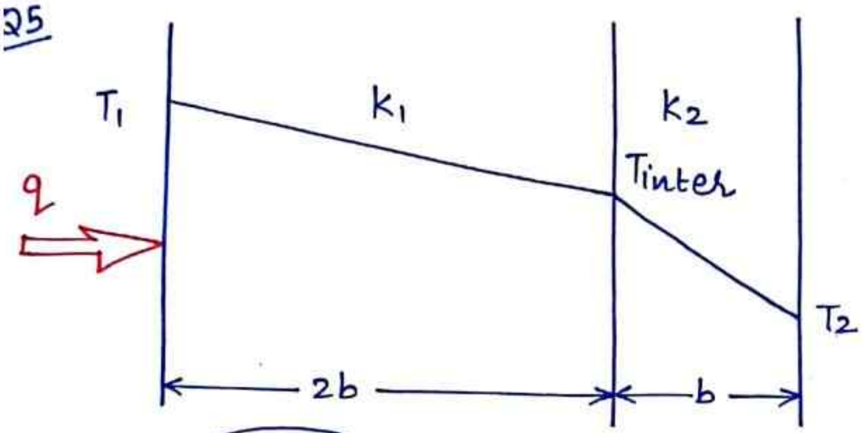




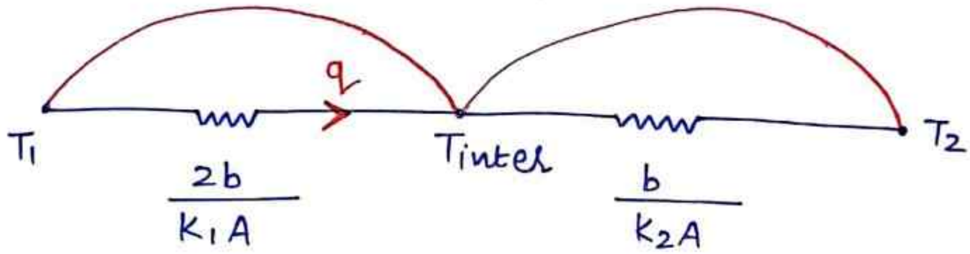
$$q = \frac{130 - T_i}{\frac{0.1}{20 \times \cancel{A}}} = \frac{T_i - 30}{\frac{0.3}{1000 \times \cancel{A}}}$$

$$T_i = 67.5°C$$

25



$k_1 > k_2$

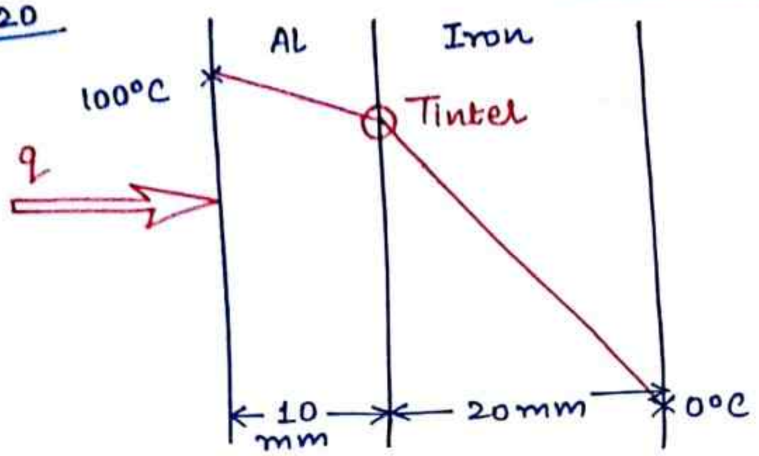


$$q = \frac{\cancel{T_1 - T_{inter}}}{\frac{2\cancel{b}}{K_1 \cancel{A}}} = \frac{\cancel{T_{inter} - T_2}}{\frac{\cancel{b}}{K_2 \cancel{A}}}$$

Put $T_{inter} = \left(\frac{T_1 + T_2}{2}\right)$ given

$$\Rightarrow 2K_2 = K_1$$

Q20

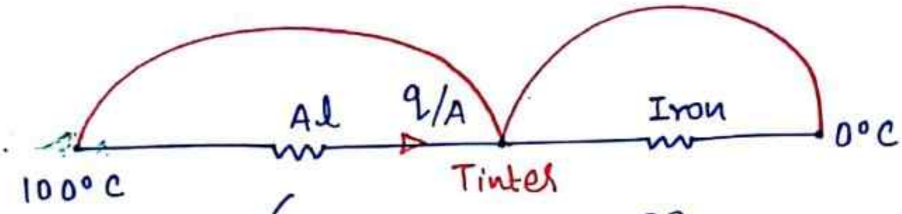


$$\frac{q}{A} = \frac{100 - T_{inter}}{\frac{10}{1000 \times K_{Al}}} = \frac{T_{inter} - 0}{\frac{20}{1000 \times K_{iron}}}$$

Put $\frac{K_{Al}}{K_{iron}} = 3$

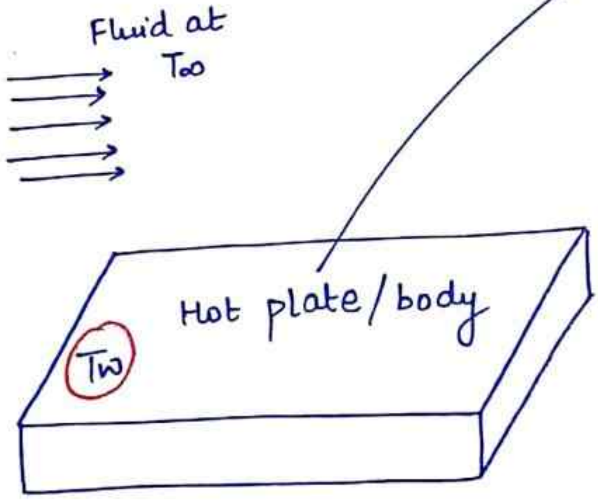
$\therefore T_{inter} = 85.7°C$

Thermal circuit



$\frac{10}{1000 \times K_{Al}}$ ✓ $\qquad \frac{20}{1000 \times K_{Iron}}$

* **Convection Thermal Resistance** :-



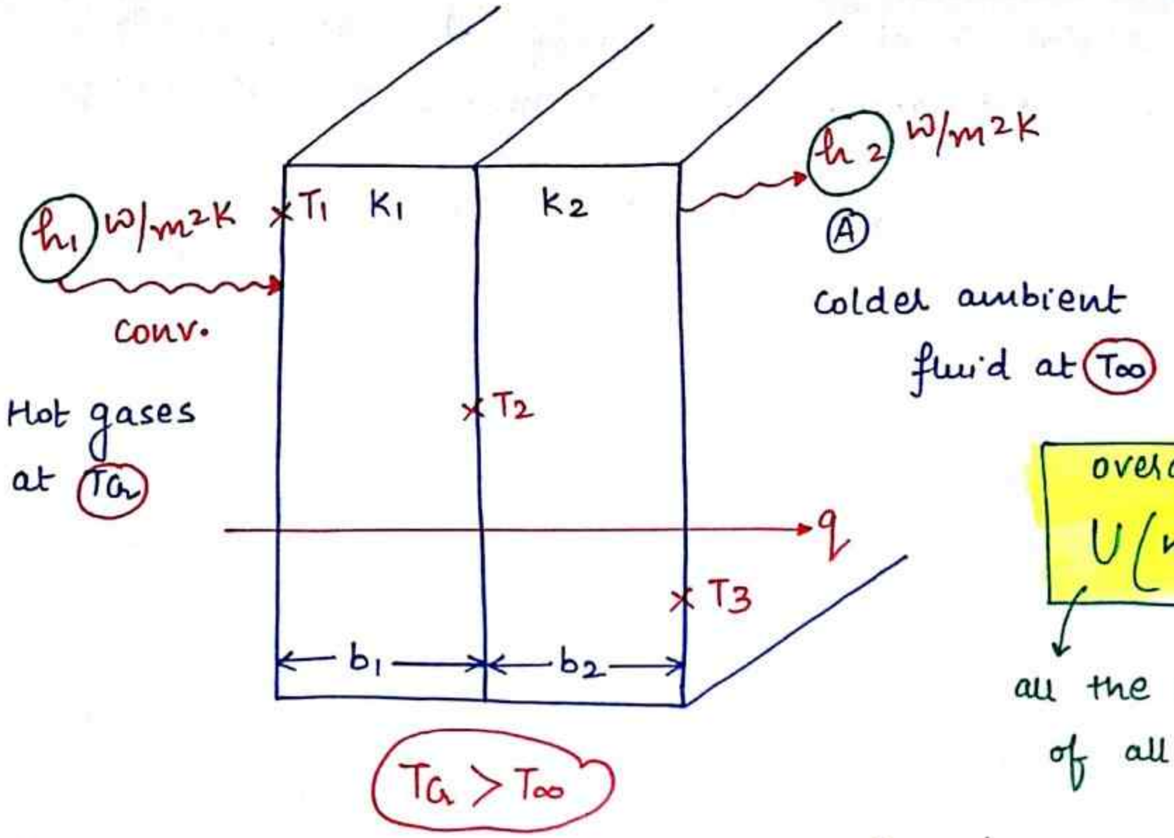
$q_{conv} = h A (T_w - T_\infty)$ — area of contact

$(R_{th}) = \frac{\Delta T}{q}$ K/watt

$\therefore (R_{th})_{conv} = \left(\frac{T_w - T_\infty}{q}\right)$

$= \left(\frac{1}{hA}\right)$ K/watt

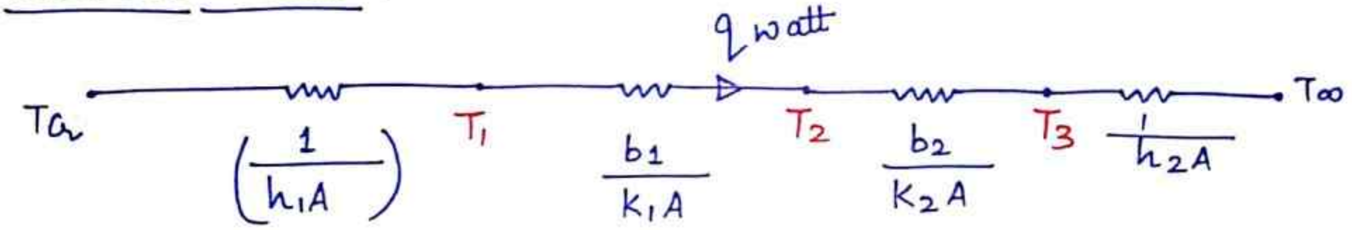
* **Conduction - Convection H.T. through a composite slab** :-



overall $U\ (W/m^2K)$ → all the Resistances of all the Modes

Assume steady state one dimensional conduction convection H.T. through composite slab b/w hot gases and the ambient colder fluid.

<u>Thermal circuit</u> :-



∴ Rate of H.T. between Hot gases and amb. fluid $= q = \dfrac{(T_G - T_\infty)\ \text{watt}}{1/h_1A + \dfrac{b_1}{K_1A} + \dfrac{b_2}{K_2A} + 1/h_2A}$

$$\Rightarrow \text{Heat flux} = q/A = \frac{(T_G - T_\infty)\ W/m^2}{\left(1/h_1 + b_1/K_1 + b_2/K_2 + 1/h_2\right)} \rightarrow (1)$$

Defining overall heat transfer coeff. U as the parameter which takes into account all the modes of heat transfer into a single entity that is

$$q = UA(\Delta T) \text{ i.e. } q = UA(T_G - T_\infty) \longrightarrow (2)$$

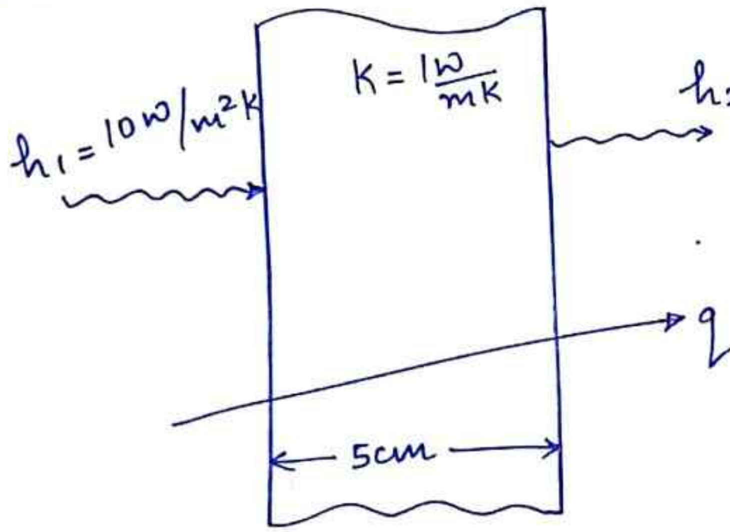
Comparing (1) & (2), we get,

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{b_1}{K_1} + \frac{b_2}{K_2} + \frac{1}{h_2}}$$

U & h have same units (W/m^2K)

NOTE :- If U value is more, the Total thermal Resistance in the entire circuit will be lesser and hence H.T. transfer rate will be higher.

Q26



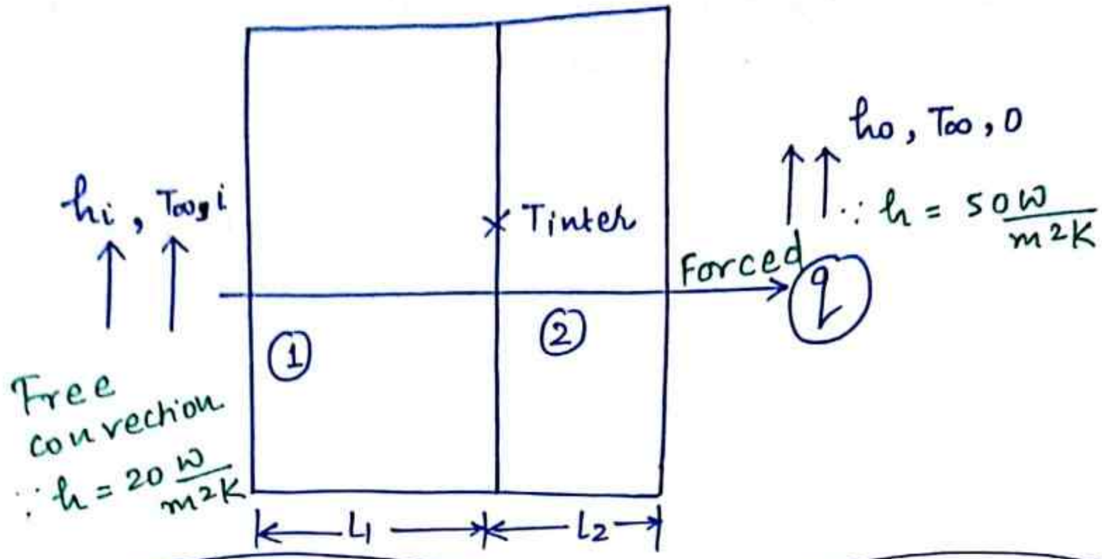
$$1/U = 1/h_1 + b/K + 1/h_2$$

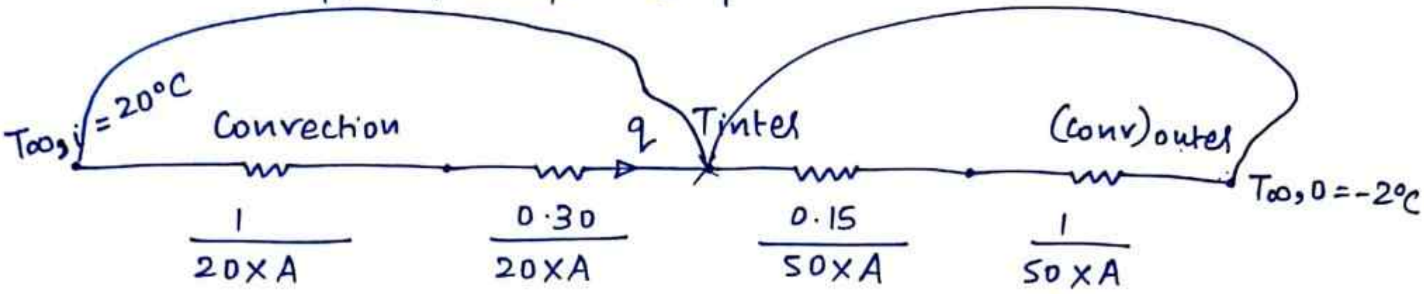
$$1/U = 1/10 + \frac{5}{100 \times 1} + \frac{1}{20}$$

$$\Rightarrow U = 5W/m^2K$$

Q6

"NEXT PAGE"

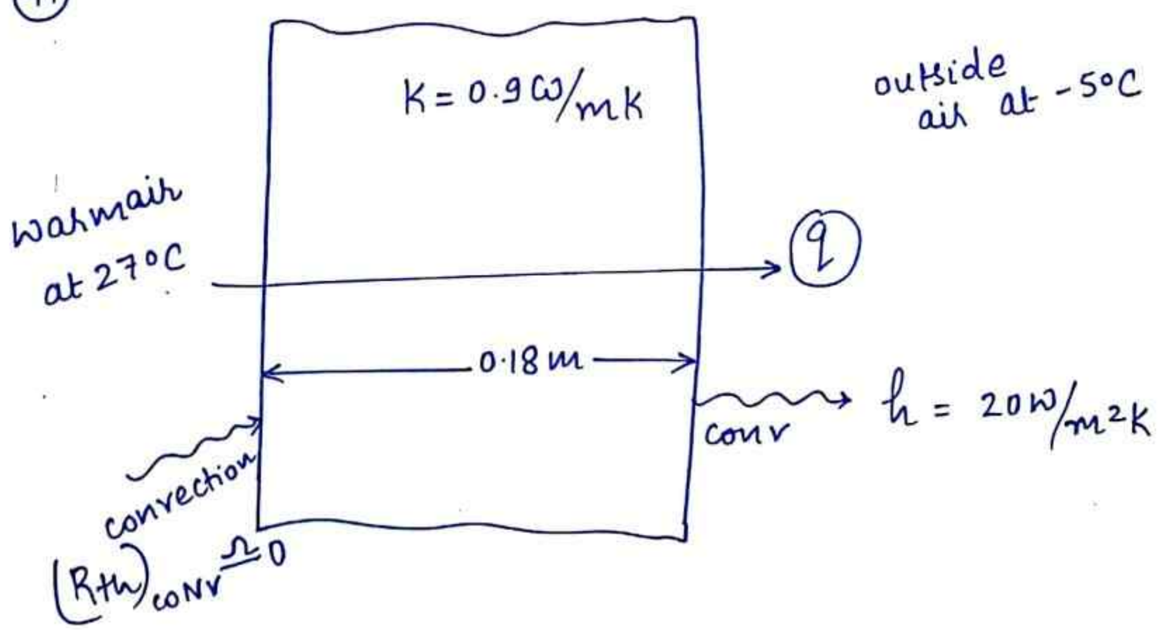




$$q = \frac{20 - T_{inter}}{\frac{1}{20\cancel{A}} + \frac{0.30}{20\cancel{A}}} = \frac{T_{inter} - (-2)}{\frac{0.15}{50\cancel{A}} + 1/50\cancel{A}}$$

$$\Rightarrow \quad T_{inter} = 3.75°C$$

41



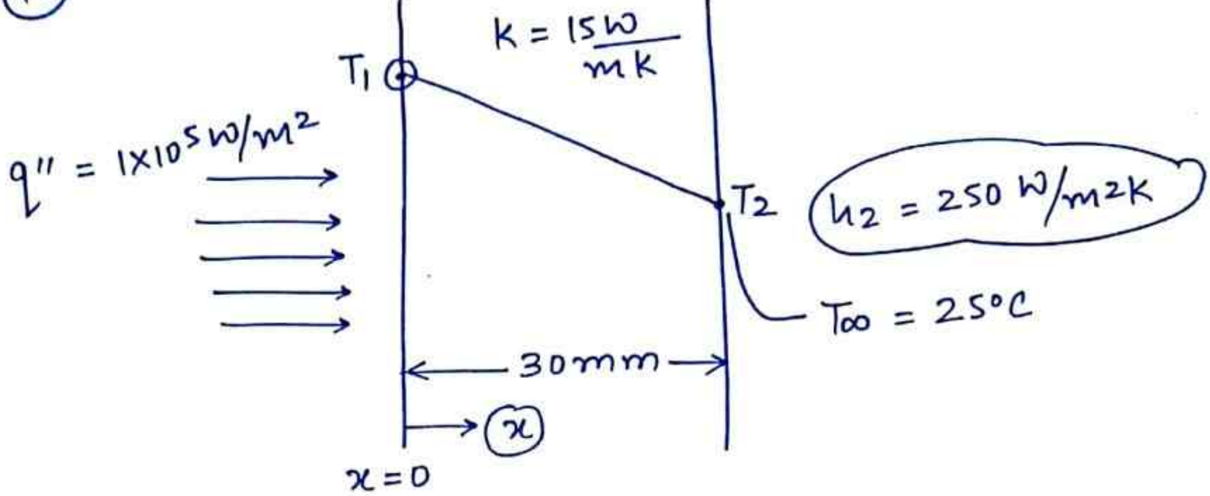
Thermal circuit

q/A W/m²

27°C — $\dfrac{0.18}{0.9\times 1}$ — $\dfrac{1}{20\times 1}$ — -5°C

$$\therefore\ q/A = \frac{27-(-5)}{\frac{0.18}{0.9} + 1/20} = 128\ \text{W/m}^2$$

4.6



Thermal Circuit :-

T_1 — Cond — $q'' = 1\times10^5$ W/m² — (conv.) — $T_\infty = 27°C - 2 = 25°C$

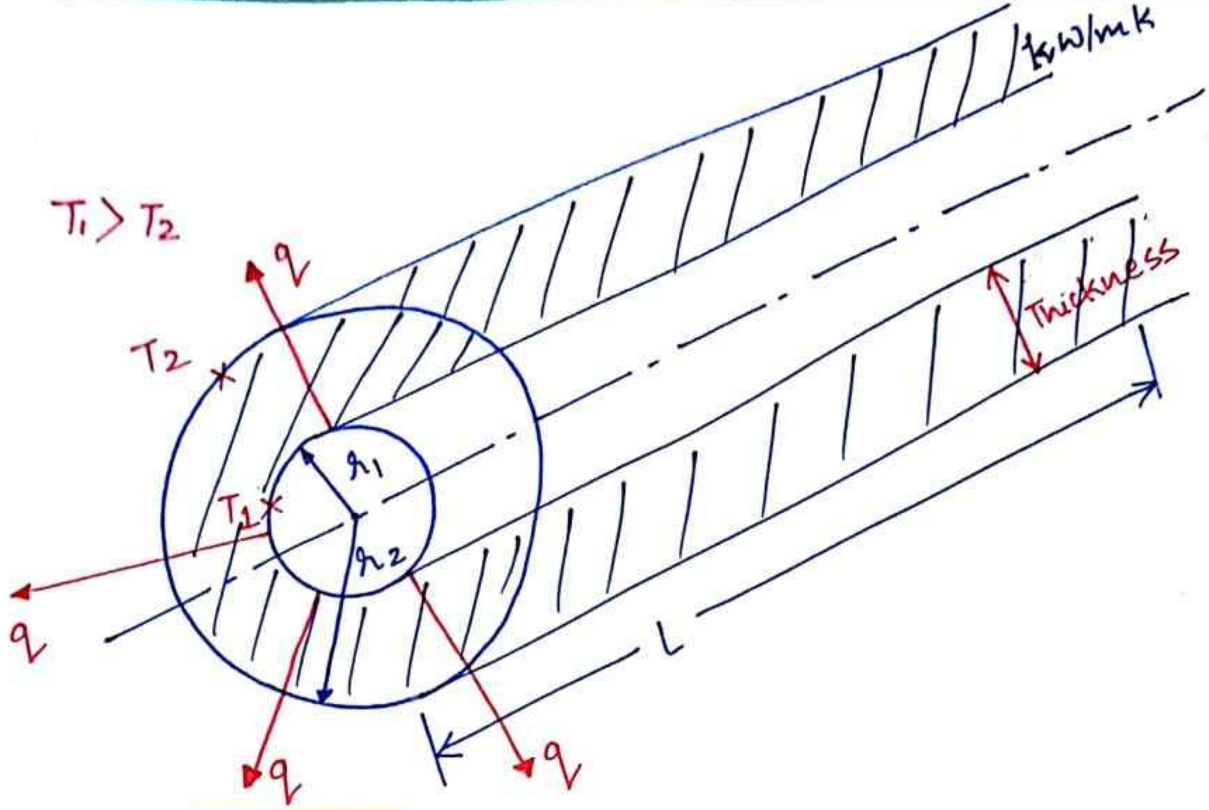
$\left(\dfrac{30}{1000\times 15\times 1}\right) \rightarrow b/KA$

$\left(\dfrac{1}{250\times 1}\right) \rightarrow 1/hA$

$$q'' = \text{Heat flux} = \frac{T_1 - 25°C}{\frac{30}{15,000} + \frac{1}{250}} = 1\times10^5\ \text{W/m}^2$$

$$T_1 = 625°C$$

* Radial Conduction H.T. through a Hollow Cylinder :-

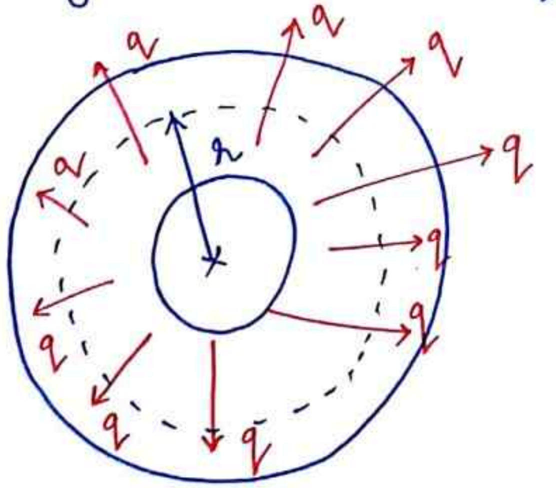


$$\boxed{T = f(r)}$$

since temperature gradients are existing along the Radial dir^n. It must conduct radially outwards from the inner cylindrical surface at T_1 to outer cylindrical surface at T_2.

Unlike in case of slabs, here the area of conduction heat transfer changes in the direction of heat flow.

At any Radius 'r', Area of conduction H.T. $= A = 2\pi r L$



$\therefore$ Fourier's law of conduction :-

Rate of Radial conduction

$$H.T. = q = -KA\left(\frac{dT}{dr}\right) \text{watt}$$

$$\Rightarrow q = -K\, 2\pi r L \left(\frac{dT}{dr}\right)$$

At $r = r_1 \Rightarrow T = T_1$

At $r = r_2 \Rightarrow T = T_2$

Assume: steady state, one dimensional (Radial) H.T.

$$\Rightarrow \int_{r_1}^{r_2} q \frac{dr}{r} = \int_{T_1}^{T_2} -2\pi K L \, dT$$

To satisfy the steady state H.T. conditions,

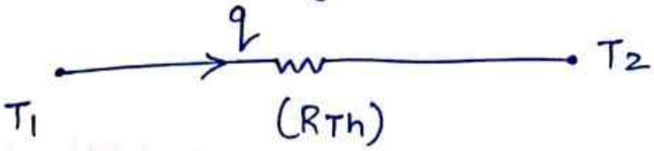
$q \neq f(r)$

i.e. $q_r = q_{r+dr}$

$$\Rightarrow q \ln\left(\frac{r_2}{r_1}\right) = 2\pi K L (T_1 - T_2)$$

$\Rightarrow$ Rate of Radial conduction H.T. $= \boxed{q = \frac{2\pi K L (T_1 - T_2)}{\ln(r_2/r_1)} \text{ (watt)}}$

The corresponding conduction thermal Resistance is



$$\underbrace{(R_{Th})_{(cond)}}_{\text{hollow cylinder}} = \left(\frac{T_1 - T_2}{q}\right) = \frac{\ln(r_2/r_1)}{2\pi K L} \text{ K/watt}$$

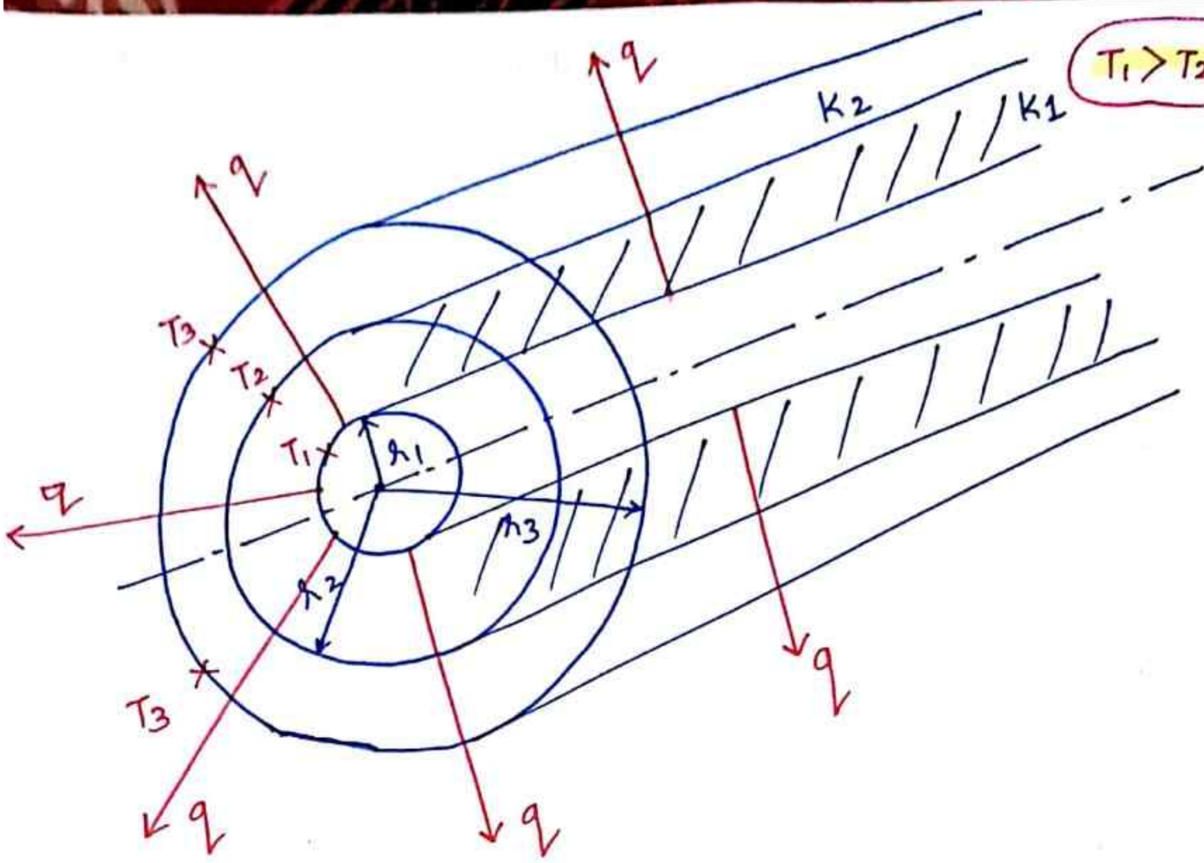
$\boxed{\text{NOTE}}$:-

If the thickness of the cylinder is very small and if the conductivity of material of cylinder is very high (a very thin copper cylinder) then the above conduction thermal Resistance almost becomes zero.

* **Radial Conduction heat Transfer through a composite cylinder:-**

" N. P. "

$T_1 > T_2 > T_3$



Since tempr. gradients are existing along the radial direction, heat must conduct radially outwards from the innermost cylindrical surface at T_1 which is at a radius of r_1 to outer-most cylindrical surface at T_3 which is at a radius of r_3.

Thermal circuit

$$T_1 \;\text{---}\!\!\underset{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K_1 L}}{\text{ww}}\!\!\text{---}\xrightarrow{q} T_2 \;\text{---}\!\!\underset{\frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi K_2 L}}{\text{ww}}\!\!\text{---}\; T_3$$

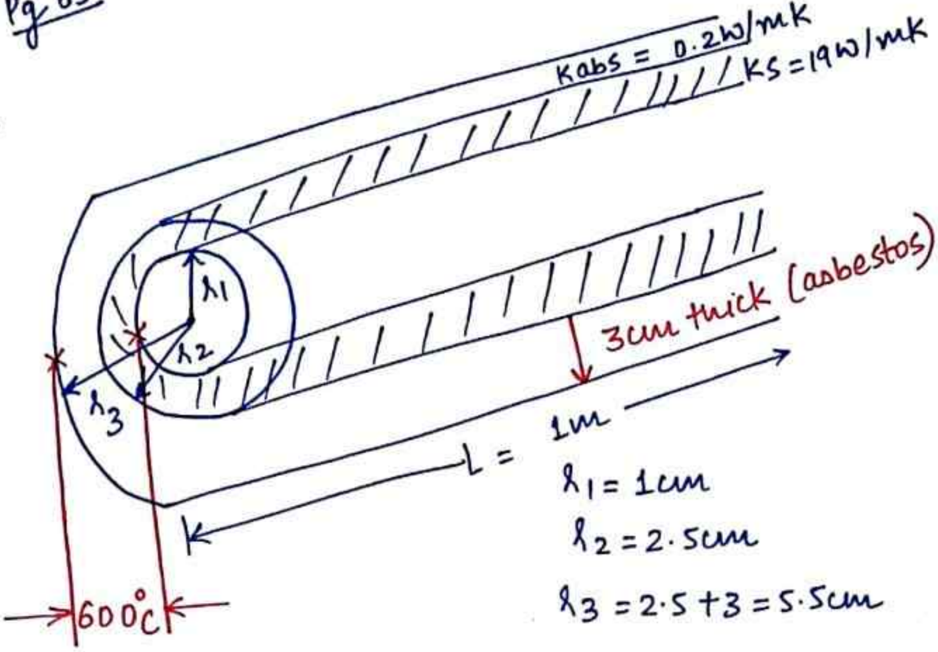
$\therefore$ Rate of Radial Conduction heat transfer (H.T.)

$$= q = \frac{(T_1 - T_3) \text{ watt}}{\frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi K_2 L}}$$

Scanned by CamScanner

WB Q2 Pg 63

Sir



$r_1 = 1cm$

$r_2 = 2.5cm$

$r_3 = 2.5 + 3 = 5.5cm$

RW

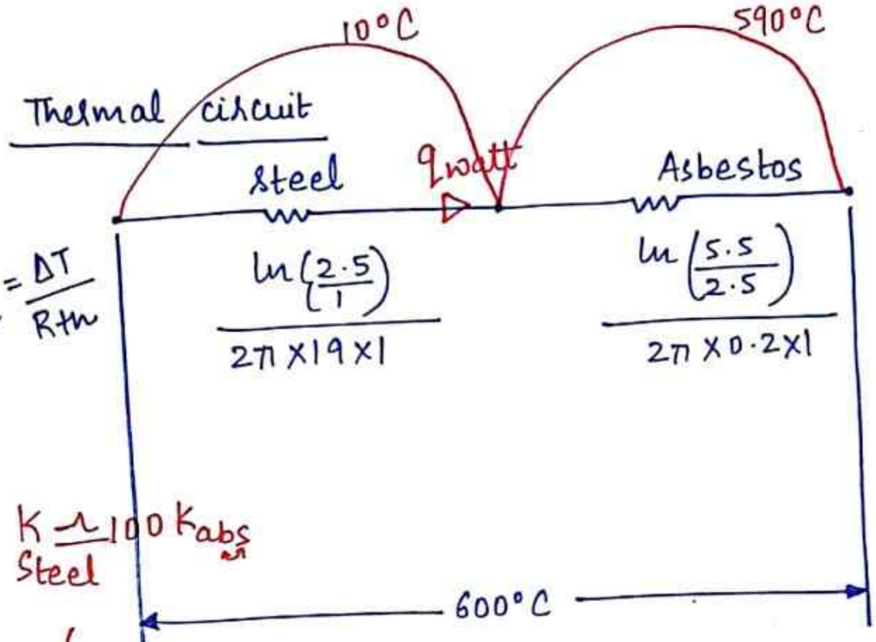
$r_1 = 1cm$

$r_2 = 2.5cm$

$r_3 = 1.5 + 1$

$= 2.$

Thermal circuit



$$q = \frac{\Delta T}{R_{th}}$$

$$\frac{\ln\left(\frac{2.5}{1}\right)}{2\pi \times 19 \times 1} \qquad \frac{\ln\left(\frac{5.5}{2.5}\right)}{2\pi \times 0.2 \times 1}$$

∴ Rate of Radial conduction H.T.

$$\Rightarrow q = \frac{600}{\Sigma R_{th}}$$

$$= 944.7 \ W/m$$

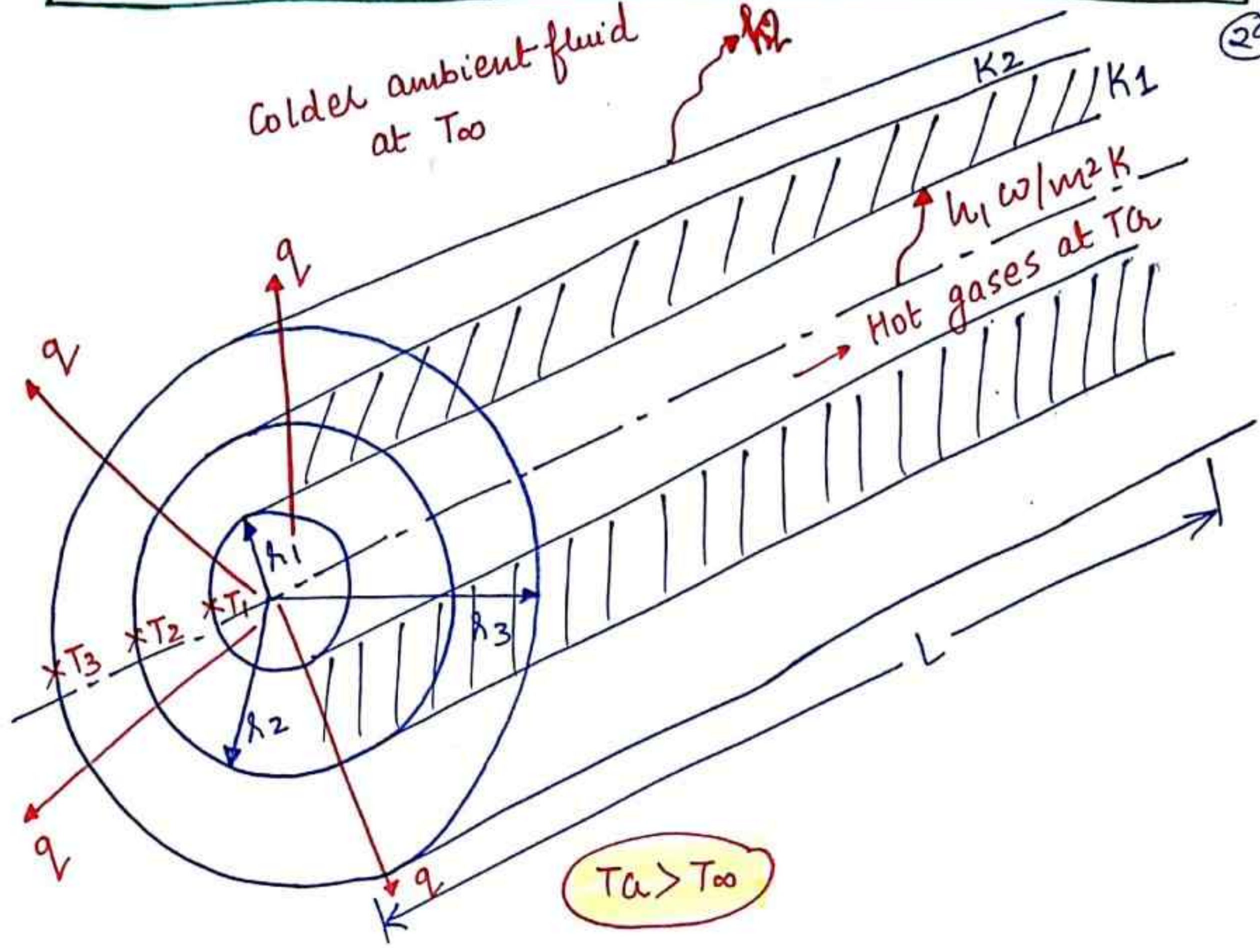
$K_{Steel} \sim 100 \, K_{abs}$

↓

$(R_{Th})_{steel} <<< (R_{Th})_{Asb}$

$\Rightarrow \boxed{(\Delta T)_{across\ steel} <<< (\Delta T)_{across\ Asb.}}$

* Radial Conduction- Convection H.T. through a composite cylinder :-



h_1 = Inside convective H.T. coefficient

h_2 = outside convective H.T. coefficient

assume steady state one dimensional Radial heat transfer b/w the hot gases and the ambient cold fluid through composite cylinder.

Thermal circuit :-

$$T_a \xrightarrow[\frac{1}{h_1(2\pi r_1 L)}]{\text{(conv) inner}} T_1 \xrightarrow[\frac{\ln(r_2/r_1)}{2\pi K_1 L}]{q\ \text{watt} \quad \text{Cond}} T_2 \xrightarrow[\frac{\ln(r_3/r_2)}{2\pi K_2 L}]{\text{Cond}} T_3 \xrightarrow[\frac{1}{h_2(2\pi r_3 L)}]{\text{(conv.) outer}} T_\infty$$

$$\therefore \text{Rate of Radial H.T.} = q = \frac{2\pi r_1 L(T_a - T_\infty)\,\text{watt}}{2\pi r_1 L\left(\frac{1}{h_1 2\pi r_1 L} + \frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi K_2 L} + \frac{1}{h_2 (2\pi r_3 L)}\right)} \longrightarrow (1)$$

Defining overall H.T. coeff. U_i that is based on the inside convection heat transfer area and overall H.T. coeff. U_o that is based on the outside convection H.T area. from the eqn :-

$$\Rightarrow q = U_i A_i \Delta T = U_o A_o \Delta T$$

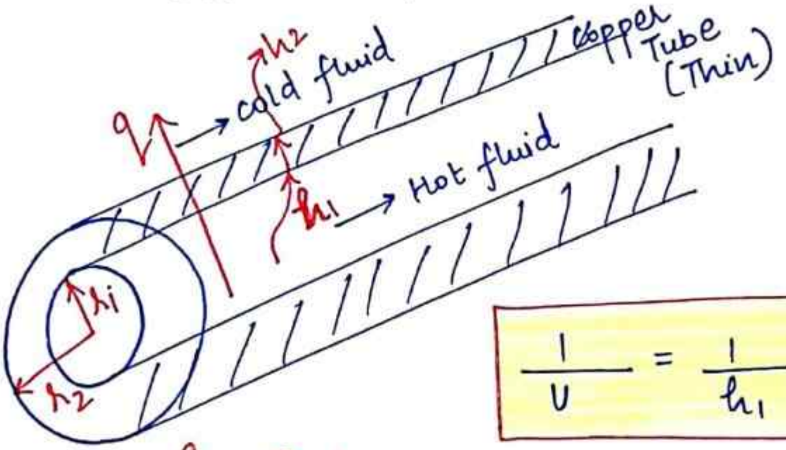
$$\Rightarrow q = U_i \underbrace{(2\pi r_1 L)}(T_a - T_\infty) = U_o \underbrace{(2\pi r_3 L)}_{\to (2)}(T_a - T_\infty) \text{ watt}$$

Comparing (1) and (2), we get

$$\boxed{\frac{1}{U_i} = \frac{1}{h_1} + \frac{r_1}{K_1} \ln\left(\frac{r_2}{r_1}\right) + \frac{r_1}{K_2} \ln\left(\frac{r_3}{r_2}\right) + \frac{r_1}{r_3} \cdot 1/h_2}$$

$$\boxed{\frac{1}{U_o} = \frac{r_3}{r_1} \frac{1}{h_1} + \frac{r_3}{K_1} \ln\left(\frac{r_2}{r_1}\right) + \frac{r_3}{K_2} \ln\left(\frac{r_3}{r_2}\right) + 1/h_2}$$

$\boxed{\text{NOTE}}$:- But in Heat Exchanger analysis, whether LMTD method or effectiveness NTU method, U can be calculated as



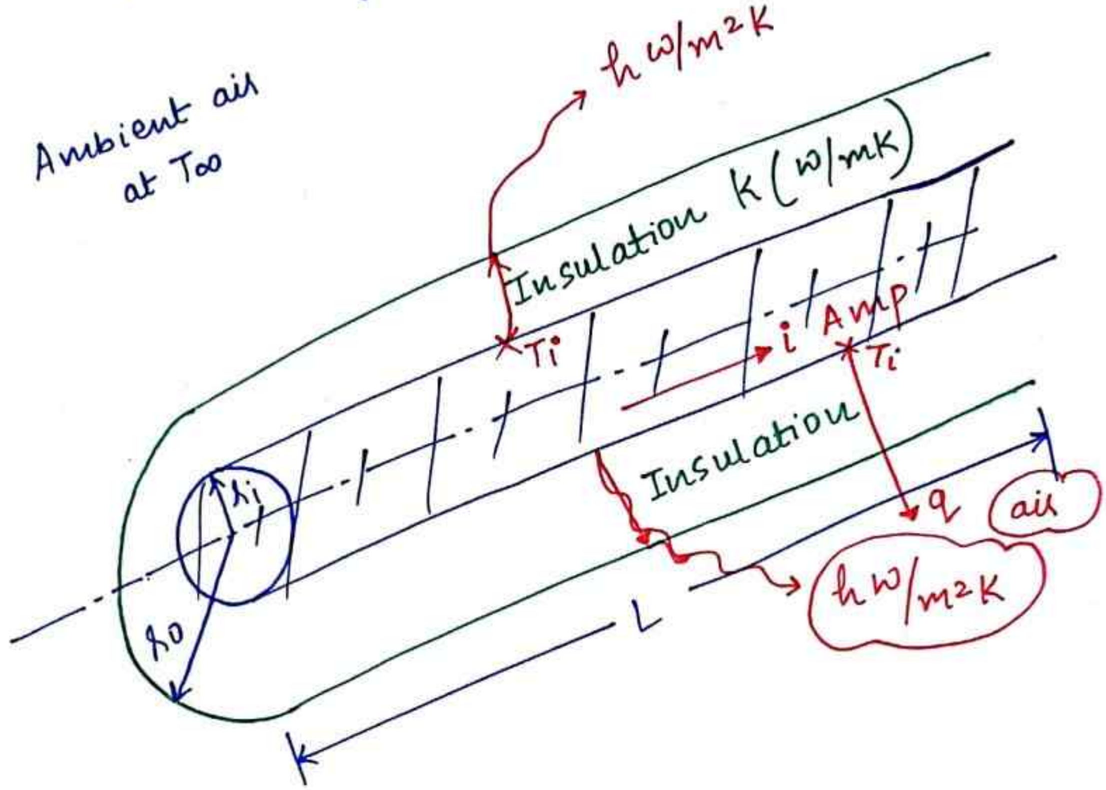
$r_i \simeq r_o$

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}}$$

(Neglecting conduction Thermal Resistance of Tube wall)

* Critical Radius of Insulation

Note:- for sufficiently thin wires, putting the insulation around the wire may result in increase of heat transfer rate instead of decreasing it.



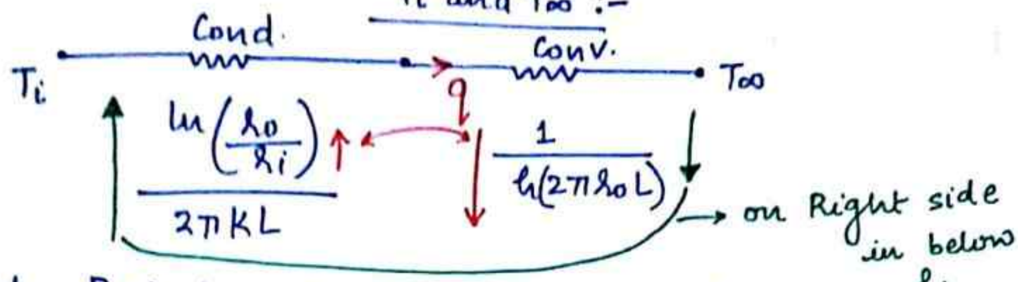
Consider a solid wire of radius r_i inside which heat is being generated by passing electric current. Let an insulation having a thermal conductivity 'K' is been wrapped around the wire upto the radius r_o.

The heat generated in the wire ~~is radially~~ due to passage of current is radially conducted through the insulation and then from the surface of the insulation heat is convected to the ambient fluid at T_∞ with a convective heat transfer coefficient of h W/m²K

Assuming steady state H.T. conditions,

let T_i be the surface tempr. of wire.

Drawing Thermal circuit for Radial H.T. b/w T_i and T_∞ :-



$\therefore$ Rate of Radial H.T. between wire and ambient $= q = \dfrac{(T_i - T_\infty) \text{ watt}}{\dfrac{\ln(r_o/r_i)}{2\pi KL} + \dfrac{1}{h 2\pi r_o L}}$

Treating all other parameters including h as constant in the above functional relationship. Now, q becomes a fn. of r_o only.

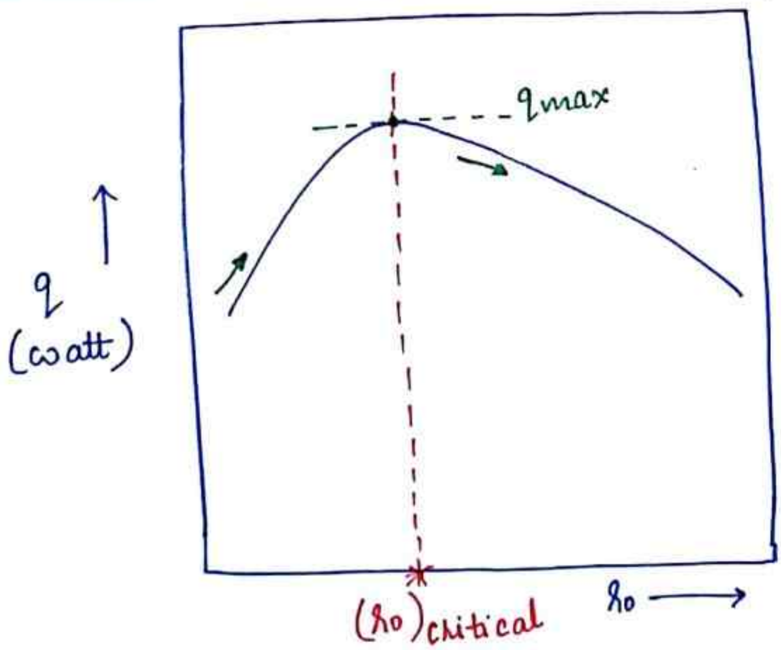
$$q = f(r_o) \text{ only}$$

For maximum H.T. rate,

$$\frac{dq}{dr} = 0 \Rightarrow \frac{d}{dr_o}\left[\frac{T_i - T_\infty}{\dfrac{\ln(r_o/r_i)}{2\pi KL} + \dfrac{1}{h 2\pi r_o L}}\right] = 0$$

$\Rightarrow r_o = \left(\dfrac{K_{Ins}}{h}\right)$ = This is called Critical Radius of Insulation.

* **Physical Significance of Critical Radius of Insulation** :-



for sufficiently thin wires whose radius is lesser than critical radius of insulation, putting the insulation around the wire will result in increase of H.T. rate instead of decreasing it. This 33
happens so because initially when more and more insulation is being wrapped around the wire, there is a rapid decrease in convection, thermal resistance as compared to little increase of conduction thermal resistance, the overall effect being decrease in Total thermal resistance and increase of H.T. rate.

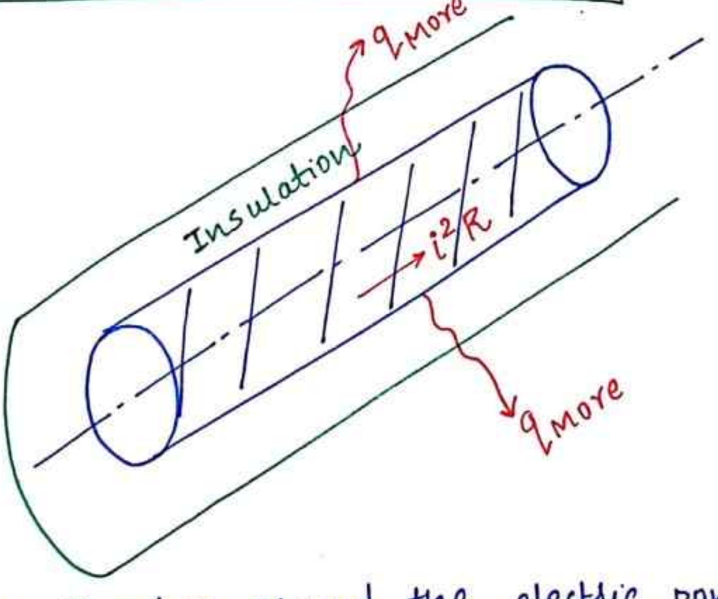
This continues upto critical radius of insulation beyond which any further insulation added shall decrease the heat transfer rate.

NOTE :- In case if the radius of the wire initially taken is already more than critical radius of insulation, any insulation wrapped around it shall directly decrease the H.T. rate.

Two Practical Applications of Critical Radius of Insulation :-

I) Electric Power Transmission Cables :-

$$r_{critical} = k/h$$

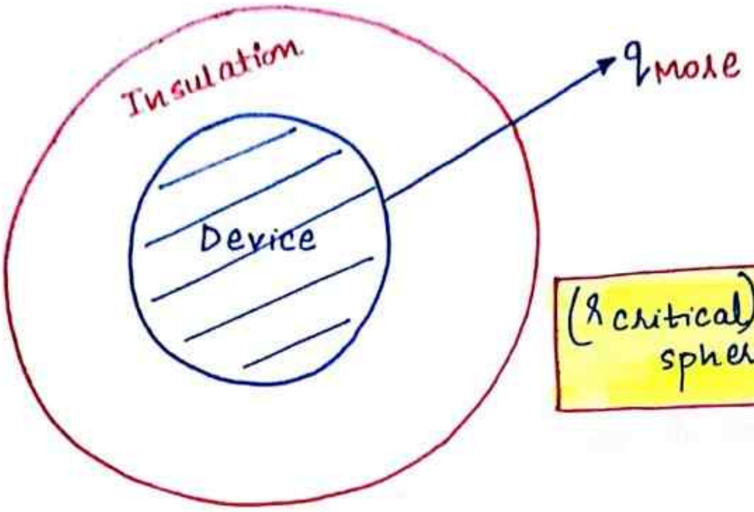


As $T_{cable} \uparrow \Rightarrow R_{electric} \uparrow$

Tempr. Resistance

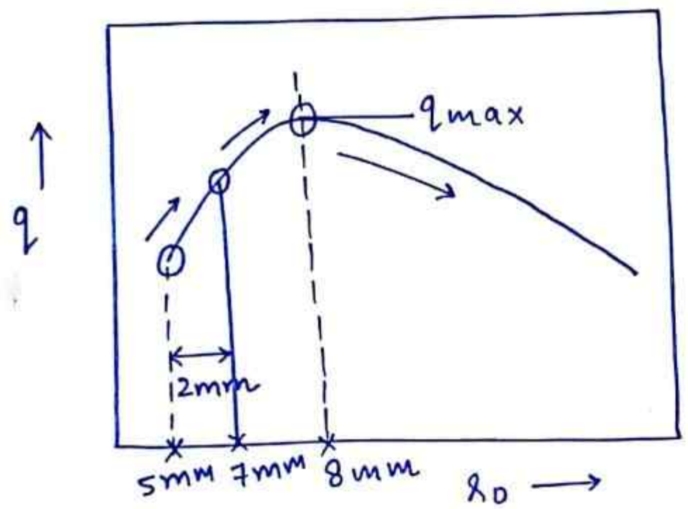
Insulation is put up around the electric power transmission cables to ↑ the H.T. Rate b/w the cable and the ambient so that the temp. of the cable can be maintained low thereby its electric resistance can be maintained low, thus transmitting more electric power.

② **Spherical electronic (semiconductor) Devices:-**



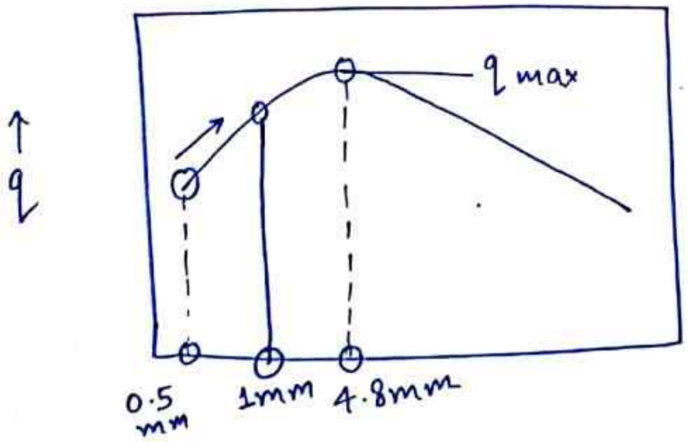
$$\boxed{(r_{critical})_{spherical} = \left(\frac{2K}{h}\right)}$$

WB ⑩ $r_{critical} = \frac{K_{INS}}{h} = \left(\frac{0.08}{10}\right) m = 8\ mm$



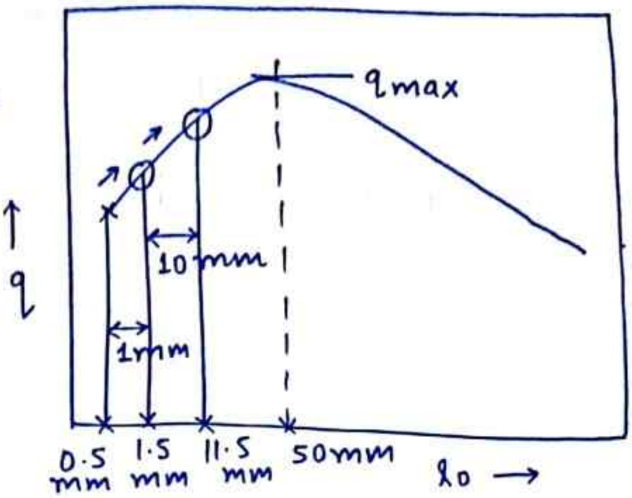
Ⓒ ✓

㊸ $r_{critical} = \frac{K_{Ins}}{h} = \frac{0.12}{(25)} = 4.8mm$



(32) $r_{critical} = \frac{K_{Ins}}{h}$

$= \left(\frac{0.5}{10}\right) = 50\text{mm}$



(a) ✓

As q from wire ↑

⇒ T_{wire} ↓ ⇒ $R_{electric}$ ↓

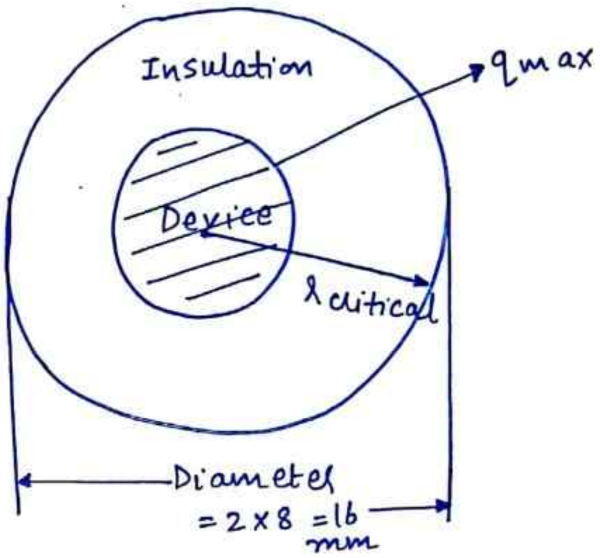
⇒ more electric Power can be transmitted

(30)

$(r_{critical})_{sph} = \frac{2K_{Ins}}{h}$

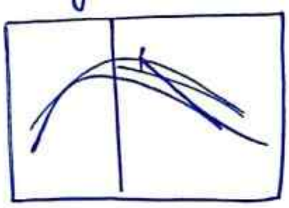
$= \frac{(0.04)}{10} \times 2$

$= 8\text{ mm}$

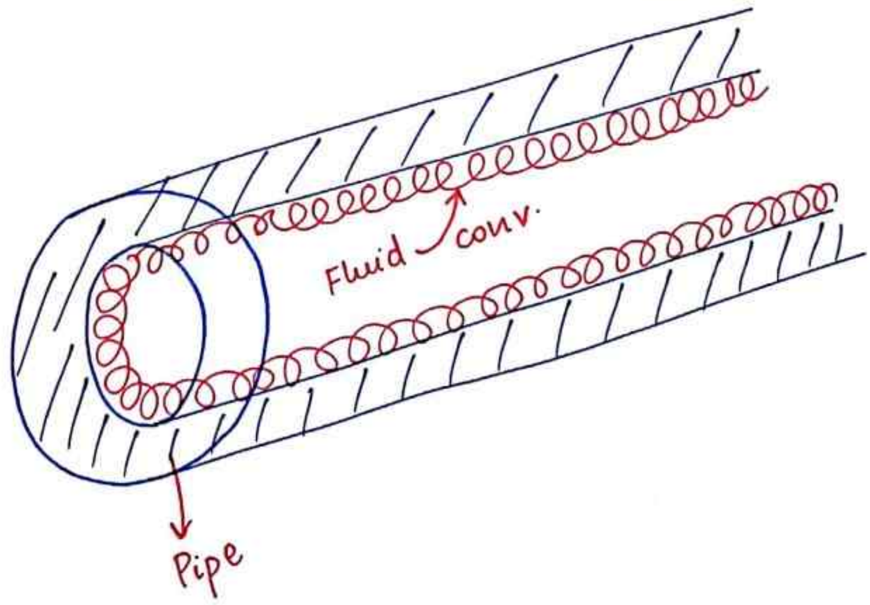


(29) c

provided the radius of cable initially taken is lesser than critical Radius of Insulation.



(42)



when Insulation is kept inside

$\Rightarrow$ convection area (or) contact area b/w fluid and pipe decreases

$\Rightarrow R_{CONV} = \frac{1}{(hA)} \uparrow$

$\therefore$ Both $R_{cond} \uparrow$ and $R_{conv} \downarrow$ $\Rightarrow$ H.T. rate always decrease

critical Radius NOT concept applicable

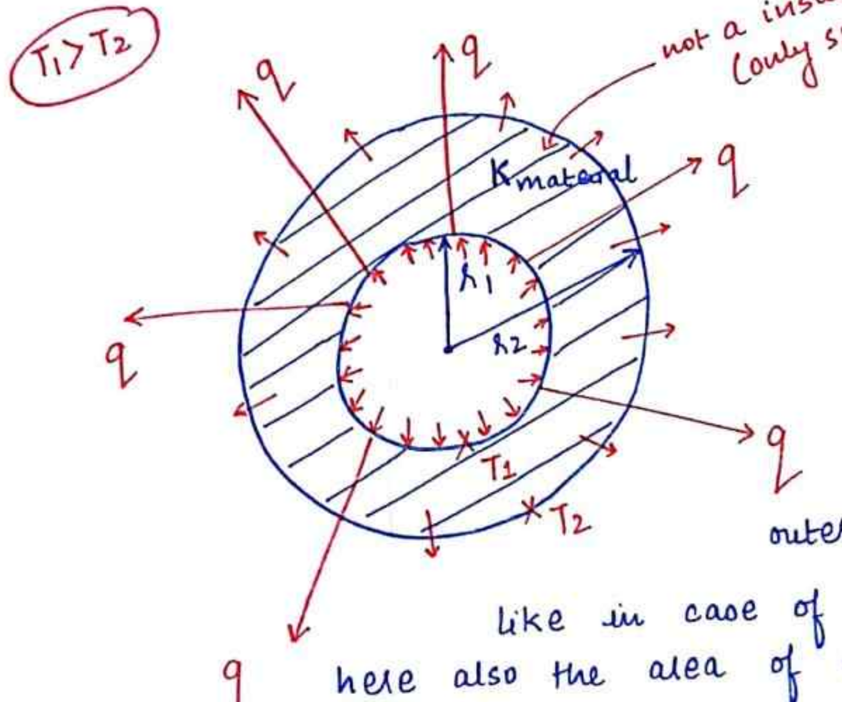
$R_{cond} \downarrow$ Rinternal, $R_{conv} \downarrow$ Rsurface

(a)

(Q 50) $r_{critical} = \frac{K_{INS}}{h_o}$ (foam)

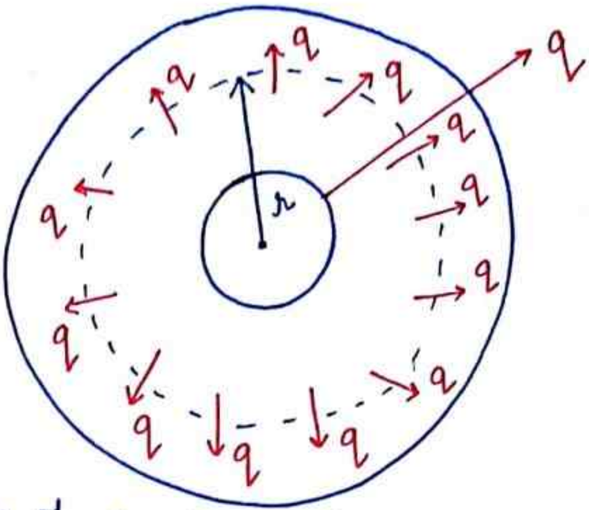
$= \left(\frac{0.1}{2}\right) m = 5 cm.$

* $\boxed{\text{Radial Conduction H.T. through a hollow sphere}}$:-



NOTE :- since tempr. gradients are existing along the radial direcn, assume steady state one dimensional conduction heat transfer between the inner spherical surface at T_1 to outer spherical surface at T_2.

like in case of cylindrical heat transfer, here also the area of conduction H.T. changes in the direction of heat flow. at any radius, 'r', area of conduction H.T. $= A = 4\pi r^2$.



Fourier's Law of Conduction :-

$\therefore$ Rate of Radial conduction $H.T. = q = -KA\dfrac{dT}{dr}$

$$\Rightarrow q = -K4\pi r^2\left(\frac{dT}{dr}\right)$$

$q \neq f(r)$ to satisfy steady state H.T. conditions $(q_r = q_{r+dr})$

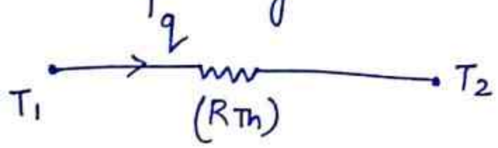
$$\Rightarrow \int_{r_1}^{r_2} q\frac{dr}{r^2} = \int_{T_1}^{T_2} -4\pi K\, dT$$

$$\Rightarrow q\left[-1/r\right]_{r_1}^{r_2} = 4\pi K(T_1 - T_2)$$

current (Thermal Potential)

$$\Rightarrow q = \frac{4\pi K(T_1 - T_2)r_1 r_2}{r_2 - r_1}$$

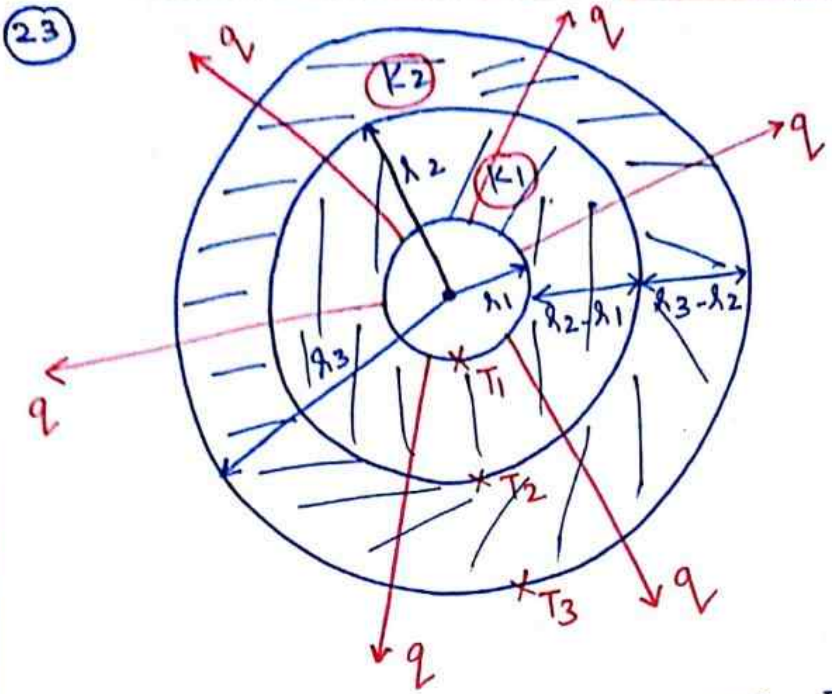
The corresponding thermal Resistance for hollow sphere is :-



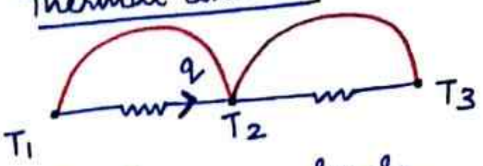
$$(R_{th})_{\text{hollow sphere (conduction)}} = \left(\frac{T_1 - T_2}{q}\right) = \frac{(r_2 - r_1)}{(4\pi K r_1 r_2)}\ K/watt$$

Scanned by CamScanner

23



Thermal circuit



$$\frac{r_2 - r_1}{4\pi k_1 r_1 r_2} \qquad \frac{r_3 - r_2}{4\pi k_2 r_2 r_3}$$

$$q = \frac{T_1 - T_2}{\frac{\cancel{(r_2 - r_1)}}{4\pi k_1 r_1 \cancel{r_2}}} = \frac{T_2 - T_3}{\frac{\cancel{(r_3 - r_2)}}{4\pi k_2 \cancel{r_2} r_3}}$$

$$\Longrightarrow \frac{T_1 - T_2}{T_2 - T_3} = 2.5$$

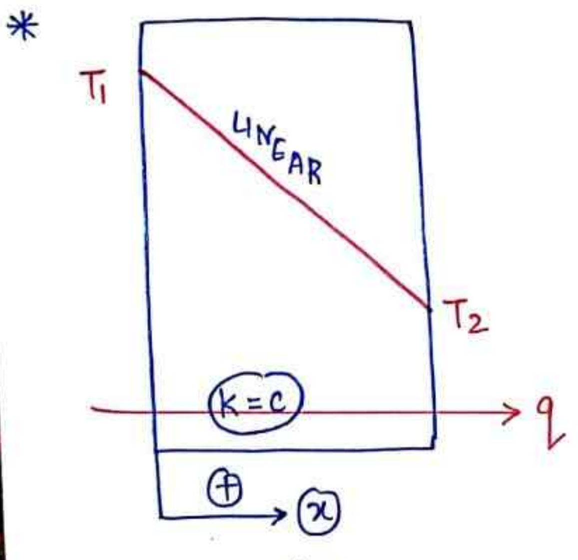
Thickness are equal

$$(r_2 - r_1) = (r_3 - r_2)$$

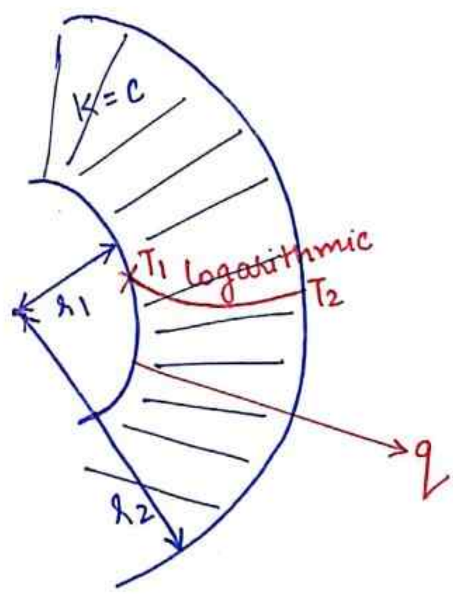
$$\frac{k_1}{k_2} = 1/2$$

$$\frac{r_1}{r_3} = 0.8$$

*

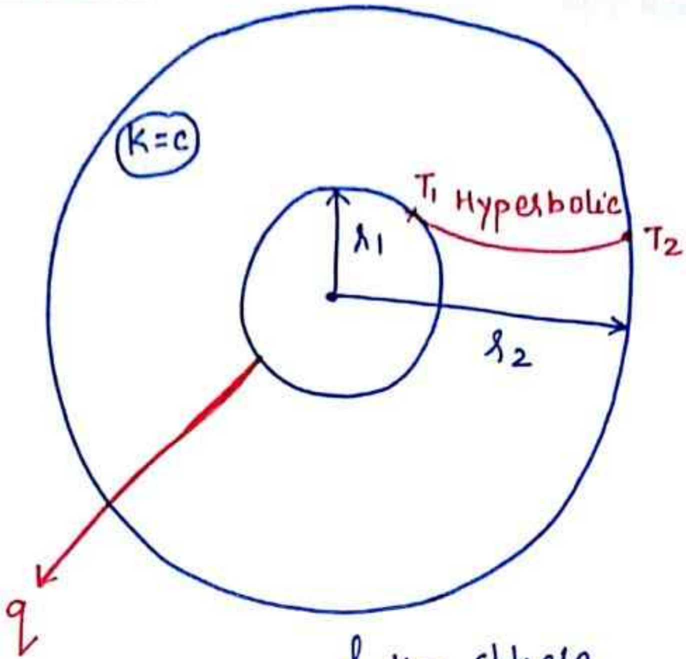


slab



hollow cylinder

*

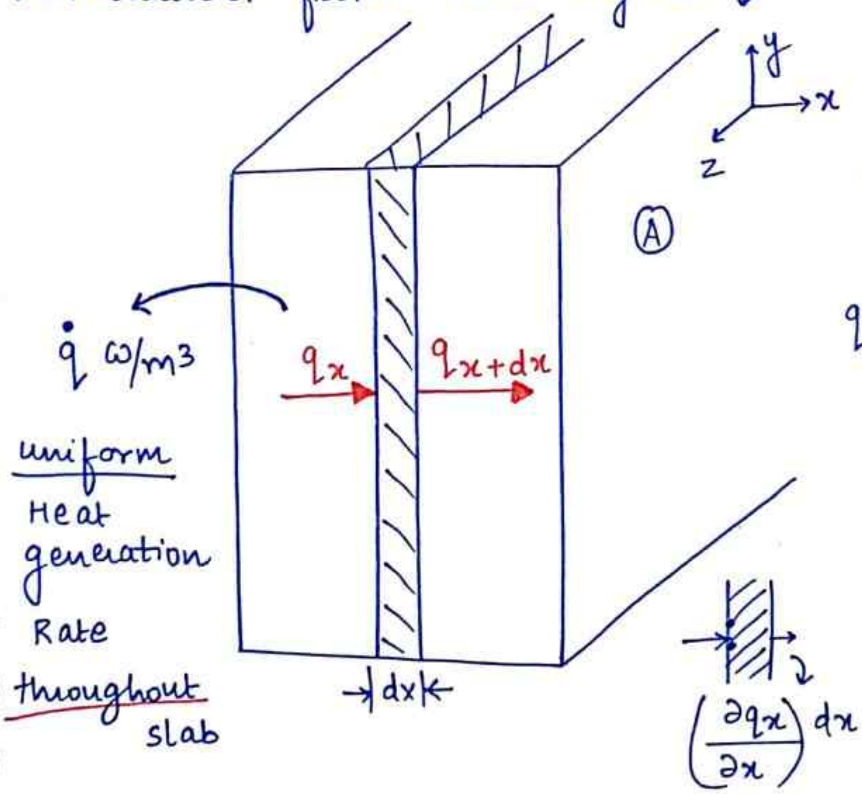


$$R_{Th} = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

hollow sphere

* **Generalised (3D, steady or Unsteady, with or without heat generation) Conduction equation :-**

Note :- heat can be generated inside a solid body either by supplying electric power or by exothermic chemical Rxn. or by thermonuclear fission. Such diagram :-



Let q_x = Heat conducted into the element along x - direction

$$q_x = -KA\left(\frac{dT}{dx}\right) \text{ watt}$$

q_{x+dx} = Heat conducted out of the element along x - direction

$$= q_x + \frac{\partial}{\partial x}(q_x)\, dx \text{ watt}$$

Heat generated in the element = $\dot{q} A\, dx$ watt

Writing Energy balance for x- direction conduction through the element,

$$q_x + q_{generated} = q_{x+dx} + \text{Rate of change of } \underset{\text{internal energy}}{\underbrace{I.E.}} \text{ of element wrt time.}$$

$$\cancel{q_x} + \dot{q}\,A\,dx = \cancel{q_x} + \frac{\partial}{\partial x}(q_x)\,dx + \frac{\partial}{\partial \tau}\underbrace{(mC_pT)}_{I.E.\text{ of element}}$$

$\tau \rightarrow$ Time in sec

where m = Mass of element

$m = \rho \times (A\,dx)$

$$\dot{q}\cancel{A}\cancel{dx} = \frac{\partial}{\partial x}\left(-K\cancel{A}\frac{dT}{dx}\right)\cancel{dx} + \frac{\partial}{\partial x}\left(\rho\cancel{A}\cancel{dx}C_pT\right)$$

But all properties (K, ρ, C_p) being constant,

$$K\frac{\partial^2 T}{\partial x^2} + \dot{q} = \rho C_p\left(\frac{\partial T}{\partial \tau}\right)$$

Writing the energy balance similarily for all the 3 Directional conductions that are occuring along x, y and z- directions simultaneously

we get $$K\frac{\partial^2 T}{\partial x^2} + K\frac{\partial^2 T}{\partial y^2} + K\frac{\partial^2 T}{\partial z^2} + \dot{q} = \rho C_p\left(\frac{\partial T}{\partial \tau}\right)$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \left(\frac{\rho C_p}{K}\right)\left(\frac{\partial T}{\partial \tau}\right)$$

Defining thermal diffusivity 'α', a thermophysical property of material as the ratio b/w the thermal conductivity of the material & its thermal capacity that is $\alpha = \left(\frac{K}{\rho C_p}\right)$ $K \rightarrow W/mK$, $\rho \rightarrow kg/m^3$, $C_p \rightarrow J/kgK$, $\alpha \rightarrow \frac{m^2}{sec}$

$(\rho C_p) \rightarrow$ heat capacity (OR) heat storage ability.

$\boxed{\text{NOTE}}$:- α of a medium or material signifies the ability of the material to allow the heat energy to ~~kept di~~ get diffused or pass through the material more rapidly.

If the α of a material is more, if either thermal conductivity of material is more or if the heat capacity of the material is lesser. (41)

$\alpha_{gases} > \alpha_{liquids}$

Ex:- $\alpha_{air} > \alpha_{water}$

$K_{air} < K_{water}$

$(\rho C_p)_{air} <<< (\rho C_p)_{water}$ .... Note over ......

Now, derivation continues

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)}$$

If conditions are steady, then $\frac{\partial T}{\partial \tau} = 0$

$T \neq f(\tau)$

and If there is no heat generation, $\dot{q} = 0$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$$\Rightarrow \boxed{\nabla^2 T = 0}$$

Laplace equ. in T.

JB Q14

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial t}\right)$$

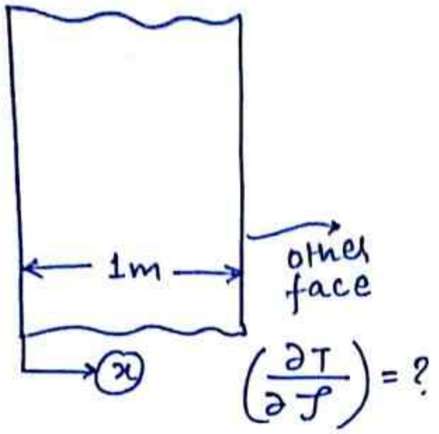
$t \rightarrow$ Time

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

But $\alpha = c$ (constant properties)

$$\frac{\partial T}{\partial t} \propto \frac{\partial^2 T}{\partial x^2}$$

Q27



$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{(\partial \tau)}$$

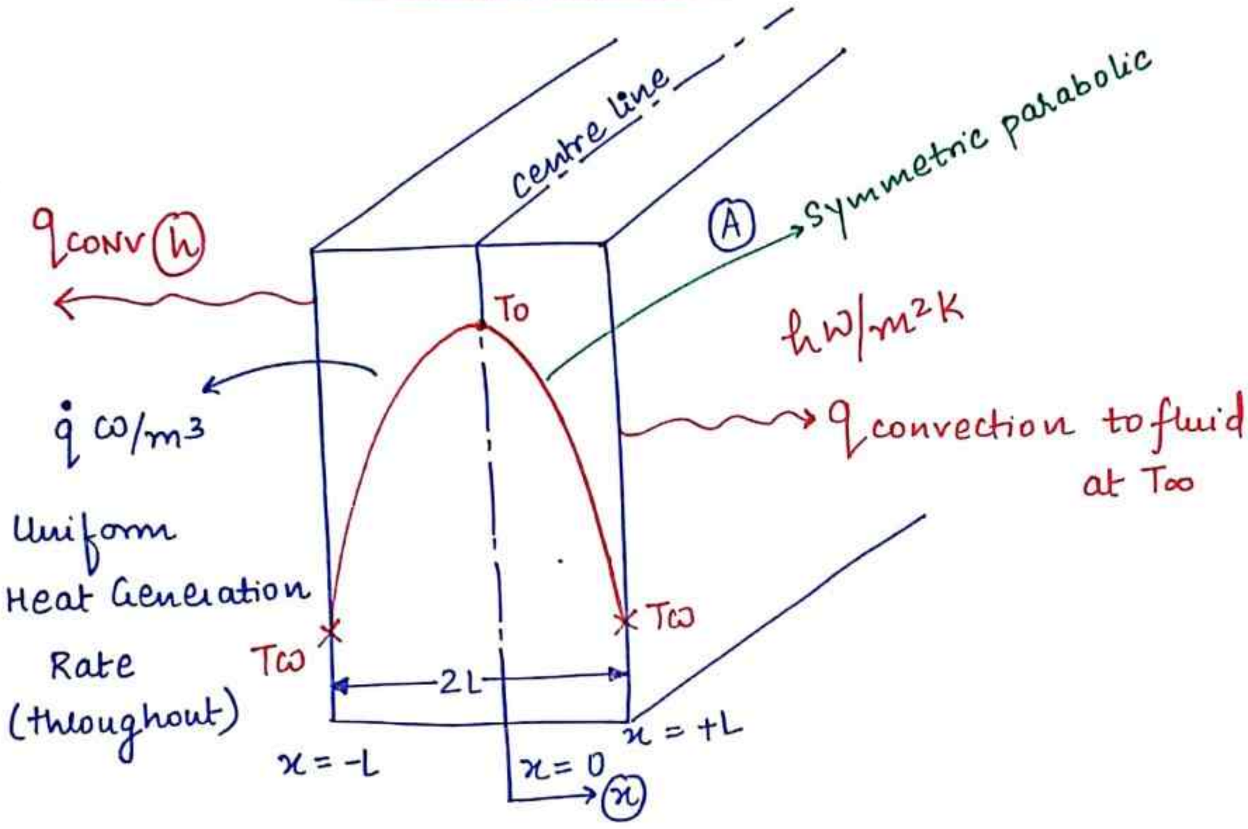
$$\frac{\partial T}{\partial \tau} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial T}{\partial \tau} = \alpha (40 + 60x)$$

Put $x = 1m$

$$\Rightarrow \left(\frac{\partial T}{\partial \tau}\right)_{at\ x=1m} = 2 \times 10^{-3} (100)$$
$$= 0.2\ °C/Hr$$

* HEAT GENERATION IN A SLAB :-



To get Temp. distribution within the slab

Assume :- ① steady state H.T. conditions ($T \neq f(\text{time})$)

To maintain this steady state conditions of the slab while generating heat, all the heat generated in the slab must be convected to a fluid either from one side of the slab or from both the sides.

② One-Dimensional heat conduction ($T = f(x)$) only. (43)

③ Uniform heat generation rate and constant 'k' of material.

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{0} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\cancel{\frac{\partial T}{(\partial \tau)}}^{0}$$

1D 1D steady

$$\Rightarrow \frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \Rightarrow \frac{d^2T}{dx^2} = -\frac{\dot{q}}{k}$$

Integrating, $\frac{dT}{dx} = \frac{-\dot{q}}{k}x + C_1$

Integrating, $T = \frac{-\dot{q}}{k}\frac{x^2}{2} + C_1 x + C_2$

C_1 and C_2 are constants of $\int$ that are to be obtained from boundary conditions.

One Special boundary Condition is :-

At $x = +L$ and $x = -L \Rightarrow T = T_w$

i.e. both sides of slab are subjected to same Temp.

$\boxed{\text{NOTE}}$:- This can be possible only if both sides of the slab are subjected to the same fluid at the same tempr. with the same convective H.T. coefficient h. To satisfy this Boundary condn, C_1 must be zero.

More over........

The Tempr. of slab is max^m.,

when $\frac{dT}{dx} = 0$

$$\Rightarrow \frac{-\dot{q}}{k}x + \cancel{C_1}^{0} = 0$$

$\Rightarrow x = 0$ i.e. we see max^m. Tempr. of slab at its centreline (or Mid-plane) of slab.

Note :- In case if both sides of the slab are at different tempr's. then C_1 would not be zero which means we would not see the maxm. tempr. at the mid-plane of the slab.

..over . . . .

Let the maxm. Tempr of slab be T_0.

$$\therefore \text{ At } x = 0 \Rightarrow T = T_0 \quad \left\{ \because \cancel{T} = -\cancel{\frac{\dot{q}}{k}\frac{x^2}{2}} + \cancel{C_1 x} + C_2 \right\}$$

$$\Rightarrow C_2 = T_0$$

Therefore the tempr. distribution within the slab is :-

$$T = -\frac{\dot{q}}{K}\frac{x^2}{2} + T_0$$

$$\Rightarrow \boxed{T_0 - T = \frac{\dot{q}\,x^2}{2K}} \quad \text{— (1)}$$

Parabolic Tempr. distribution.

At $x = +L$ (or) $x = -L \Rightarrow T = T_\omega$

$$T_0 - T_\omega = \frac{\dot{q}L^2}{2K} \quad \text{— (2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{T_0 - T}{T_0 - T_\omega} = \left(\frac{x}{L}\right)^2$$

Non-dimensional format of Tempr. distribution

The sidewall tempr. T_ω can be obtained from energy Balance eqn. for a steady state conditions of the slab that is heat generated in the slab = Heat convected from the slab to fluid

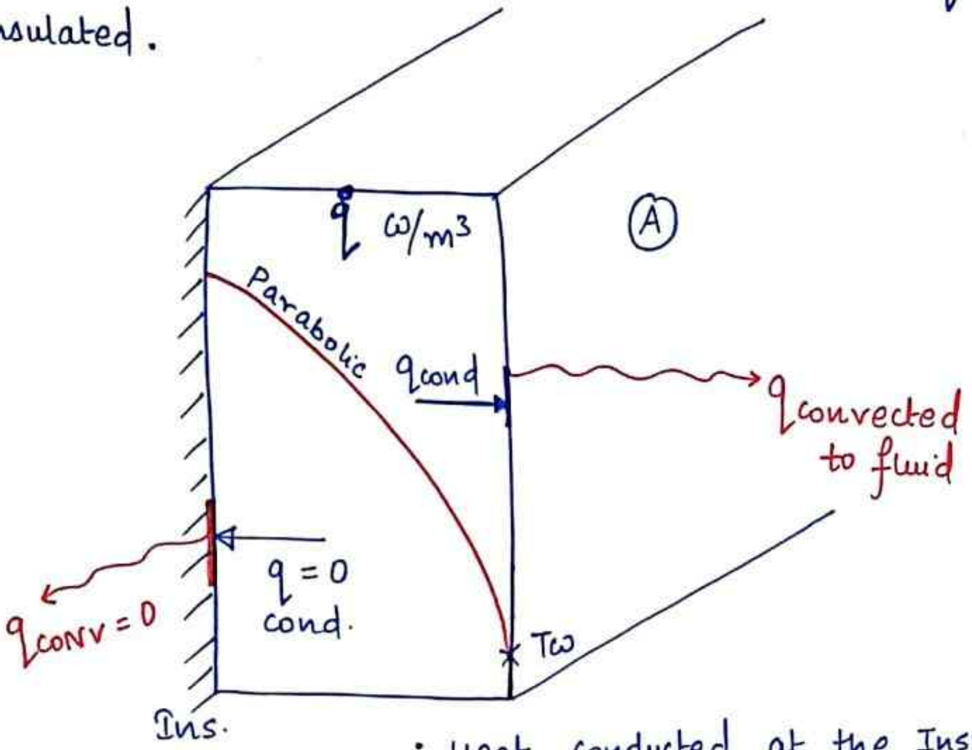
$$\Rightarrow \dot{q} \times (\cancel{2}L \times \cancel{A}) = \cancel{2}h\cancel{A}(T_\omega - T_\infty) \text{ watt}$$

$$\Rightarrow \boxed{T_\omega = \frac{\dot{q}L}{h} + T_\infty}$$

$$\Rightarrow T_0 \text{ (or) } T_{max} = \frac{\dot{q}L^2}{2K} + \frac{\dot{q}L}{h} + T_\infty$$

i.e. at the Mid Plane

The other extreme situation is one side of the slab perfectly insulated.

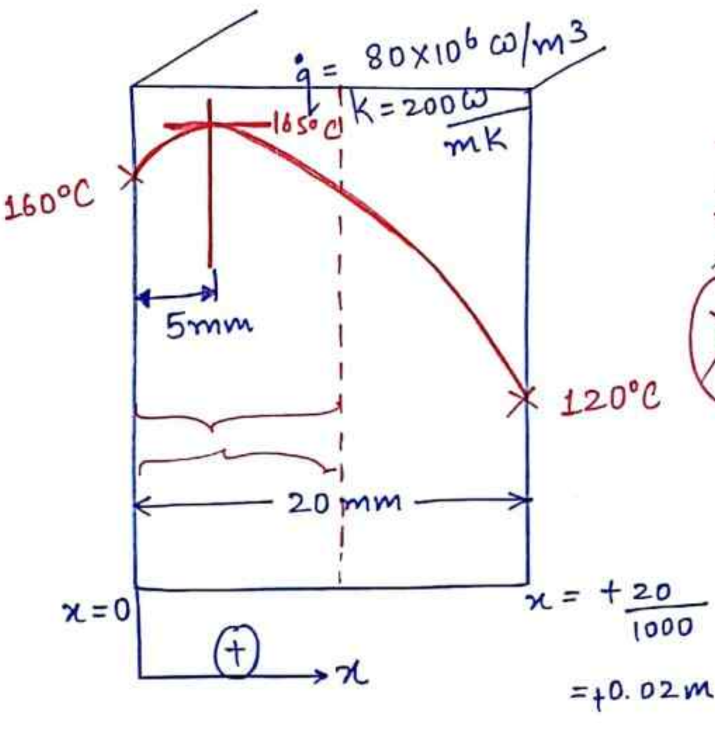


∴ Heat conducted at the Insulated surface $= 0$

$\Rightarrow -KA\left(\frac{dT}{dx}\right) = 0$ at the Insulated surface $\Rightarrow \frac{dT}{dx} = 0$ at the Insulated surface

$\Rightarrow$ T is max^m. at the Insulated surface.

WB Q17 and Q18



~~S~~ on the other side.

~~x~~ ← not symm.

S

no insulation

on the highest Tempr.

Since steady state, one dimensional heat conduction with uniform heat generation,

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$\frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1$$

$$\Rightarrow T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_1 x + C_2$$

At $x = 0 \Rightarrow T = 160°C$

$$160 = C_2$$

At $x = +0.02m \Rightarrow T = 120°C$

$$120 = \frac{-80\times10^6}{200}\frac{(0.02)^2}{2} + C_1(0.02) + 160$$

$$\Rightarrow C_1 = +2000$$

The tempr. of slab is Maximum, when $\frac{dT}{dx} = 0$

$$\Rightarrow 0 = -\frac{\dot{q}}{k}x + C_1$$

$$\Rightarrow 0 = -\frac{80\times10^6}{200}x + 2000$$

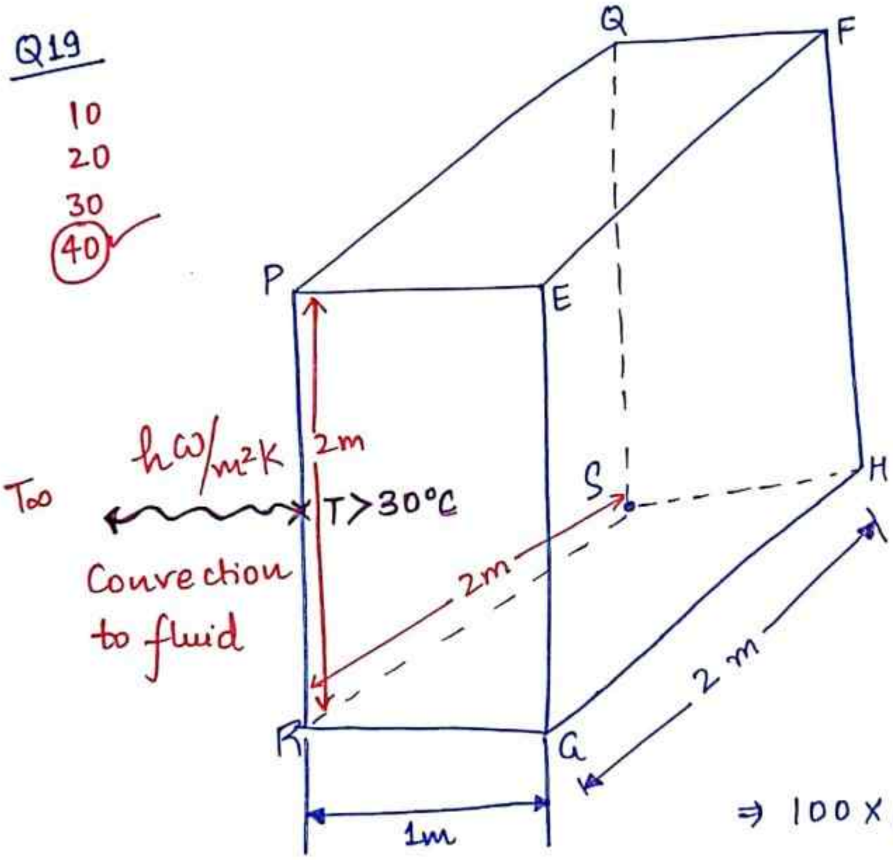
$$\Rightarrow x = 0.005\,m \text{ (location of } T_{max})$$

$$\therefore T_{max} = -\frac{80\times10^6}{200}\frac{(0.005)^2}{2} + 2000(0.005) + 160$$

$$= 165°C$$

Q19

10
20
30
40



$\therefore$ For steady state conditions of slab,

Heat generated in the slab = Heat convected from PQRS to fluid

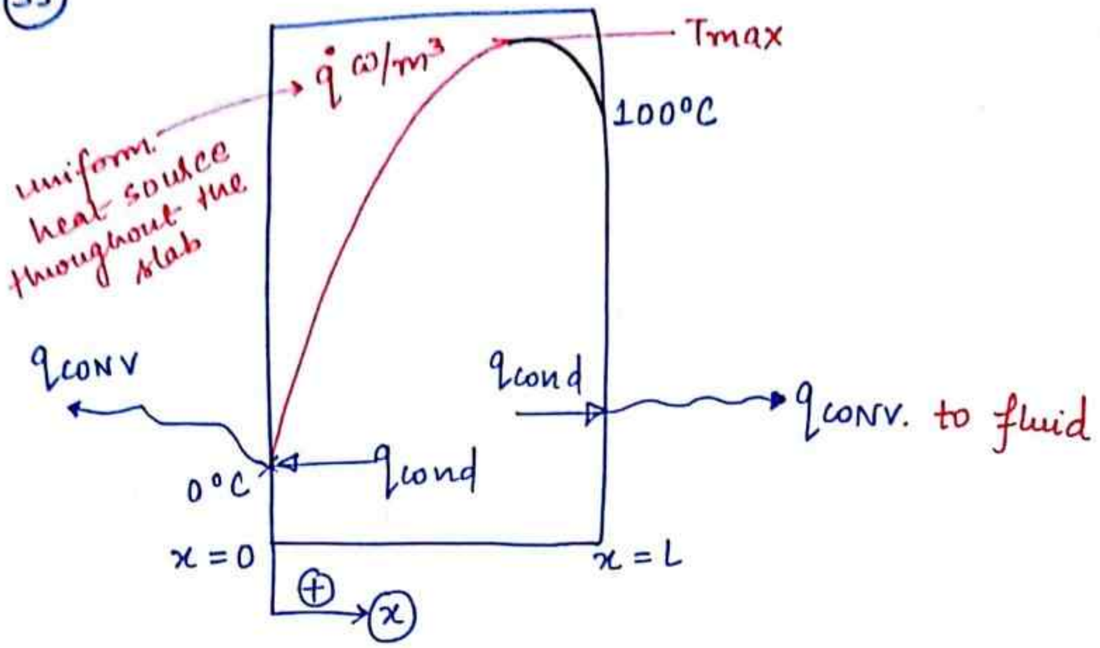
Newton's law of cooling

$$\Rightarrow \dot{q} \times \text{volume of slab} = h\,A_{PQRS}(T_{PQRS} - T_{fluid}) \text{ watt}$$

$$\Rightarrow 100 \times (2\times1\times2) \text{ watt} = 10\times2\times2(T_{PQRS} - 30) \text{ watt}$$

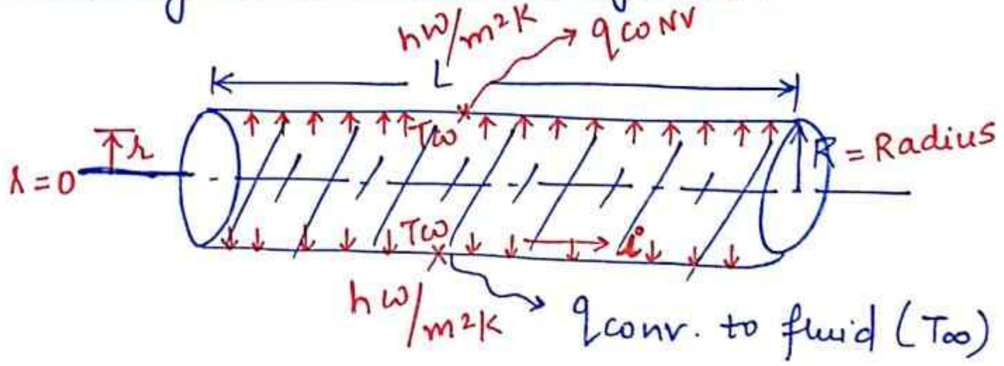
$$W/m^3 \times m^3$$

$$T_{PQRS} = 40°C$$



convection at the Boundary is always preceeded by conduction at the boundary (for steady state H.T. conditions).

* $\boxed{\text{Heat generation in the cylinder}}$:-



$\dot{q}$ = uniform Heat generation rate

$$= \frac{i^2 \cdot R_{electric}}{\pi R^2 L} \text{ watt/m}^3$$

Objective :- To get Tempr. distribution within the cylinder

i.e. $T = f(r)$

Assume :- ① steady state H.T. conditions $T \neq f(\text{Time})$.

To maintain this steady state condition of the rod while generating heat, all the heat generated in the rod must be convective to a fluid surrounding the rod with a convective H.T. coefficient of h W/m²K

② One-dimensional (Radial) conduction H.T.

$$T = f(r) \text{ only}$$

③ Uniform Heat generation Rate & constant 'K' of Material.

The Generalized Heat conduction eqn. in cylindrical coordinate system is given by :-

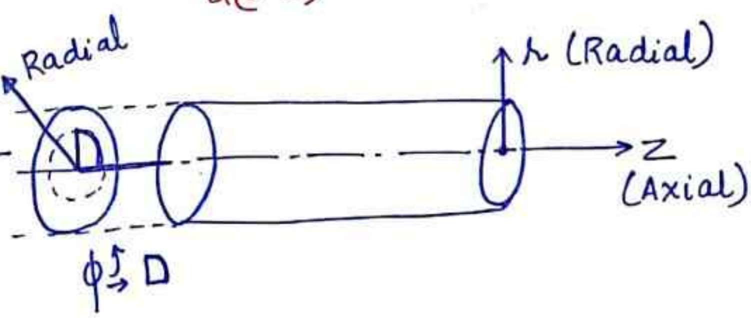
$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}}}_{1D} \rightarrow 0 + \underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{1D} \rightarrow 0 + \frac{\dot{q}}{K} = \frac{1}{\alpha}\underbrace{\cancel{\left(\frac{\partial T}{\partial \tau}\right)}}_{Steady} \rightarrow 0$$

$$\Rightarrow \frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = \frac{-\dot{q}}{K}$$

$$\Rightarrow r\frac{d^2T}{dr^2} + \frac{dT}{dr} = \frac{-\dot{q}r}{K}$$

$d(vu) = u\,dv + v\,du.$

$$\Rightarrow \frac{d}{dr}\left(\overset{u}{r}\,\overset{v}{\frac{dT}{dr}}\right) = \frac{-\dot{q}r}{K}$$



Integrating w.r.t. 'r',

$$r\frac{dT}{dr} = \frac{-\dot{q}r^2}{2K} + C_1$$

$$\Rightarrow \frac{dT}{dr} = -\frac{\dot{q}r}{2K} + \frac{C_1}{r} \Rightarrow \left(\frac{dT}{dr}\right)_{at\, r=R} = \frac{-\dot{q}R}{2K} + \frac{C_1}{R} \longrightarrow ②$$

Integrating,

$$T = \frac{-\dot{q}r^2}{4K} + C_1 \log r + C_2$$

C_1 & C_2 are constants of $\int$ that are to be obtained from boundary condn's.

Scanned by CamScanner

One boundary condition is :-

At $r = R$ (i.e. at the periphery of Rod) $\Rightarrow T = T_\omega$

The second boundary condition is :-

For steady state conditions of Rod, writting energy Balance:-

Heat generated in the Rod = Heat conducted Radially at the surface = Heat convected from surface to fluid.

$$\Rightarrow \dot{q} \times \pi R^2 L = -K(2\pi R L)\left(\frac{dT}{dr}\right)_{at\ r=R}$$

$$\Rightarrow \boxed{\left(\frac{dT}{dr}\right)_{at\ r=R} = \frac{-\dot{q}R}{2K}} \longrightarrow (1)$$

comparing eqⁿs (1) and (2),

C_1 must be zero

The tempr. of the rod is maximum, when

$$\left(\frac{dT}{dr}\right) = 0$$

$$\Rightarrow 0 = \frac{-\dot{q}r}{2K} + \cancelto{0}{\frac{C_1}{r}}$$

$\Rightarrow r = 0$ (i.e. at the axis of Rod)

i.e. we see Max. Tempr. at the axis of Rod

Let the maxᵐ. Tempʳ of Rod be T_0

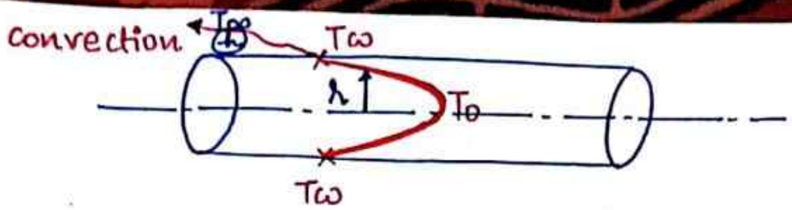
At $r = 0 \Rightarrow T = T_0$

$\Rightarrow T_0 = C_2$

∴ The tempr. distribution within the rod is :-

$$T = \frac{-\dot{q}r^2}{4K} + T_0 \Rightarrow \boxed{T_0 - T = \frac{\dot{q}r^2}{4K}} \rightarrow (3)$$

Parabolic Tempr Distribution.



At $r = R \Rightarrow T = T_\omega$

$$\therefore T_0 - T_\omega = \frac{\dot{q} R^2}{4K} \longrightarrow (4)$$

$$\frac{(3)}{(4)} \Rightarrow \frac{T_0 - T}{T_0 - T_\omega} = \left(\frac{r}{R}\right)^2$$

[Tempr. distribution in non-dimensional format]

The surface temperature T_ω can be obtained from Energy Balance Eqn. for steady state conditions of the slab i.e..

Heat generated in the Rod = Heat convected to fluid from surface.

$$\Rightarrow \dot{q} \times (\pi R^2 \cancel{L}) = h\, 2\pi R \cancel{L} (T_\omega - T_\infty) \text{ watt}$$

$$\therefore T_\omega = \frac{\dot{q} R}{2h} + T_\infty$$

surface Tempr. of rod

$$\therefore T_0 \text{ (or) } T_{max} = \left(\frac{\dot{q} R^2}{4K} + \frac{\dot{q} R}{2h} + T_\infty\right)$$

Q:> A cylindrical uranium fuel rod of radius 5.0mm in a nuclear reactor is generating heat at the rate of 4×10^7 W/m^3. The rod is cooled by a liquid (convective heat transfer coefficient is 1000 W/m^{2}K) at 25°C. at steady state, the surface tempr. in K of the rod is

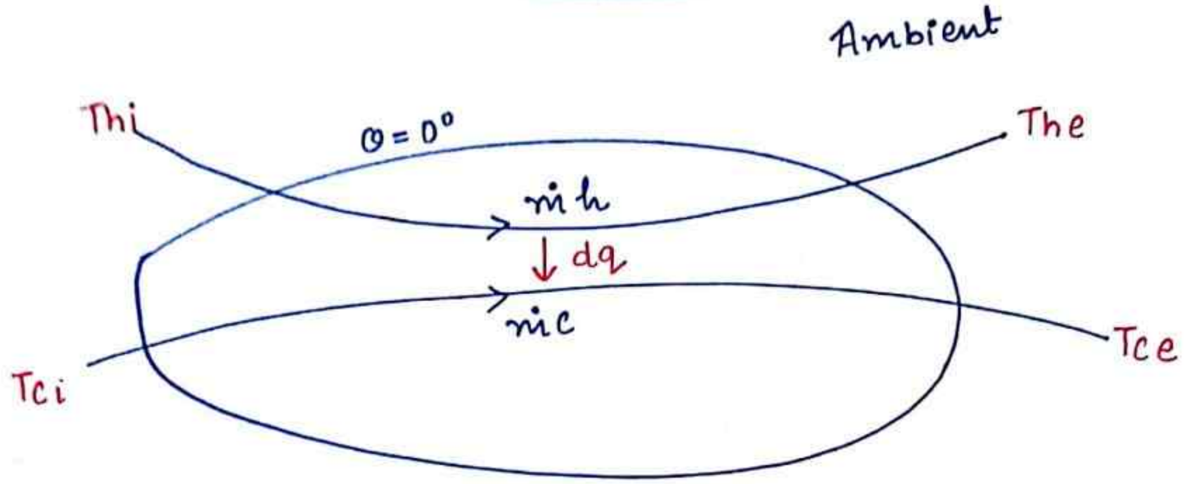
(a) 308

(b) 398 ✓

(c) 418

(d) 448

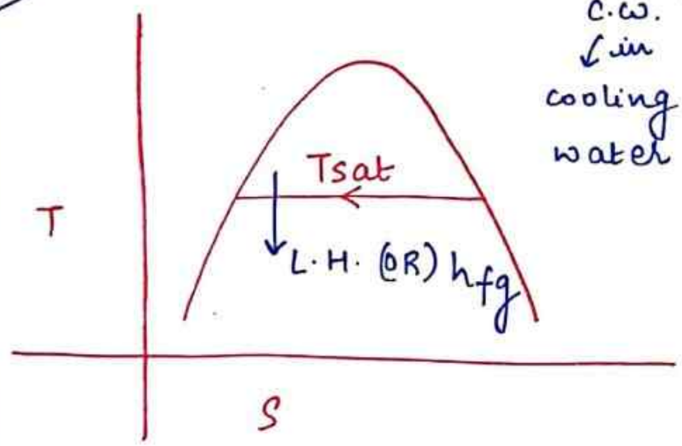
HEAT EXCHANGERS

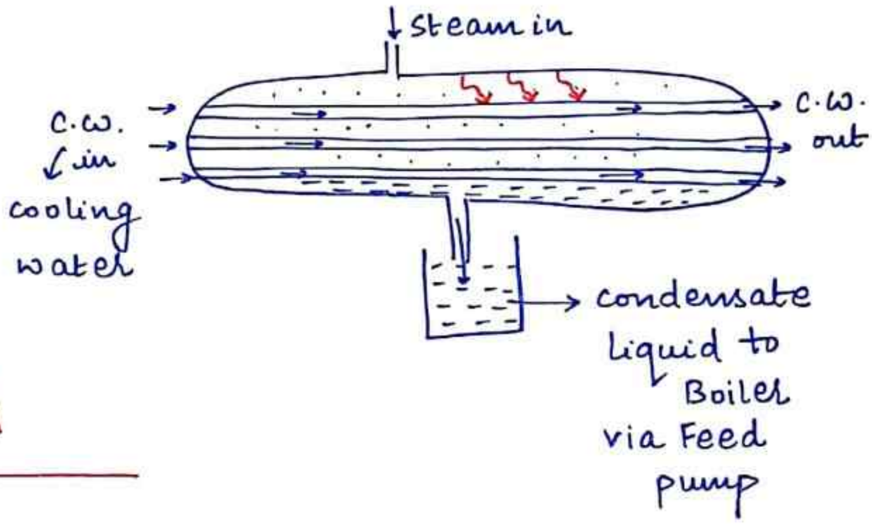


Heat Exchanger is a steady flow adiabatic open system in which two flowing fluids exchange or transfer heat b/w them without loosing or gaining any heat from the ambient.

Ex :- ① Surface (steam) Condenser (steam to C.W.)

②



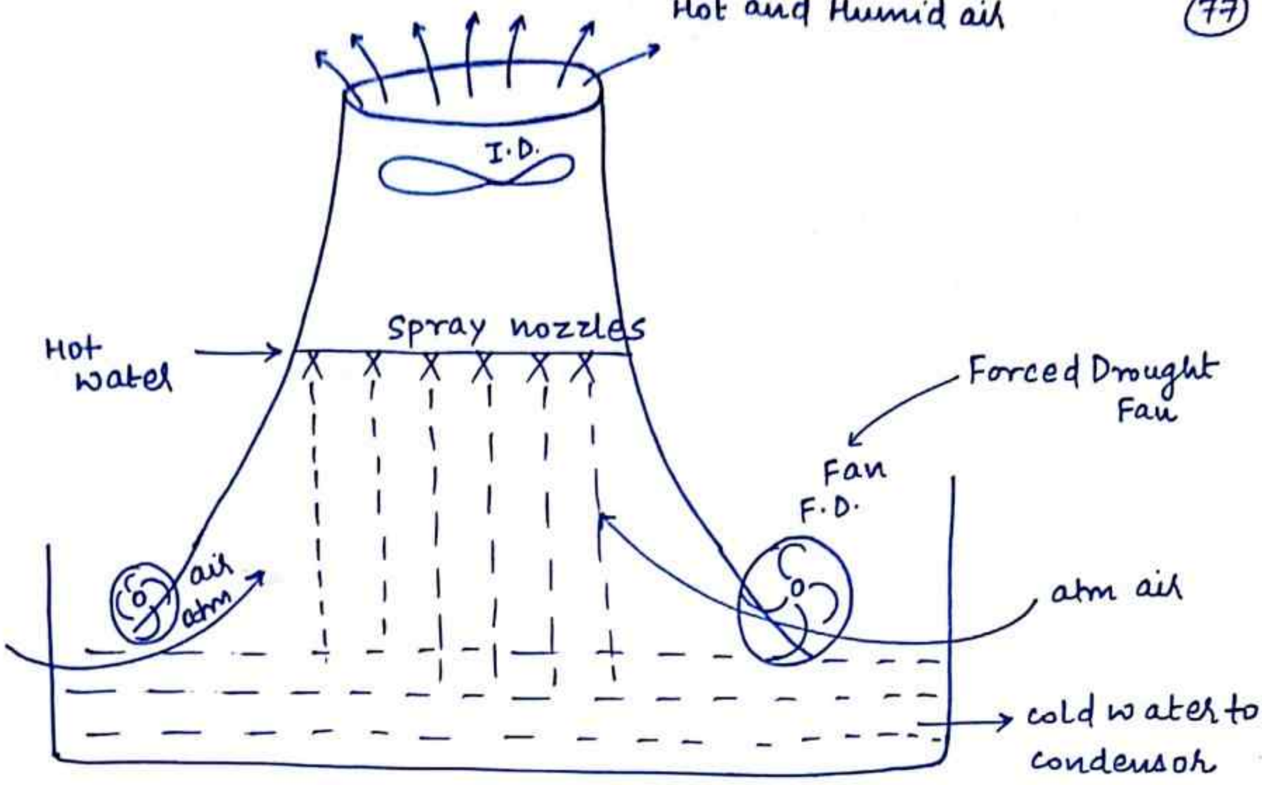


② Economiser (Hot flue gases to Feed water).

③ Superheater (Hot flue gases to Dry saturated steam).

④ Air Preheater (Hot flue gases to combustion air).

⑤ Cooling Tower (Hot water to atm. air).



⑥ Jet condenser (steam to c.w.)
coolant water

⑦ Oil cooler (Hot oil to coolant water / air).

✓ $\boxed{\text{Application of I}^{st}\text{ law of T/D to any Heat Exchanger}}$:-

Since H.E. being Steady flow open system, writting SFEE

$$\cancel{Q}^{0} - \cancel{W}^{0} = \Delta H + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0}$$

$$\Rightarrow \therefore (\Delta H)_{HE} = 0$$

$$\Rightarrow (\Delta H)_{hot\ fluid} + (\Delta H)_{cold\ fluid} = 0$$

$$\Rightarrow -(\Delta H)_{hot\ fluid} = +(\Delta H)_{cold\ fluid}.$$

Hence, $\therefore$ the rate of enthalpy decrease of hot fluid = The Rate of enthalpy increase of cold fluid.

$$\Rightarrow Q = \boxed{\dot{m}_h c_{p_h} (T_{hi} - T_{he}) = \dot{m}_c c_{p_c} (T_{ce} - T_{ci}) \text{ J/sec}}$$

Energy Balance Equ. (or) Heat Balance Equ.

$$\boxed{Q_{(P=C)} = \Delta H}$$

from T/D we know that H.T. in any constant pressure or isobaric process is equal to change in enthalpy of the fluid. also we assume that in any heat exchanger analysis, the pressures of both hot and cold fluid remains constant as they flow through heat Exchanger. By combinning above 2 points, we may conclude that the rate of heat transfer b/w hot and cold fluids in any heat exchanger is equal to the rate of enthalpy change of either of the fluids.

Rate of H.T. in entire H.E.
$$\boxed{Q = \dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci}) \text{ watt}}$$

But the precaution is that do not use the above format of the equation for calculating enthalpy change of any fluid if it is undergoing phase change like steam condensation.

* TYPES OF HEAT EXCHANGER (CLASSIFICATIONS) :-

① Direct transfer type H.E^s.

② Direct contact Type H.E^s

③ Regenerative (or) storage Type of H.E.S

① In Direct transfer type H.E's, both hot and cold fluids do not have any physical contact between them but the transfer of heat occurs between them through pipe wall of separation.

Ex :- ⓐ Surface Condenser.

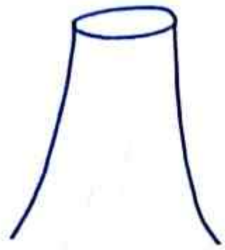
ⓑ oil cooler.

ⓒ Economiser.

ⓓ Airpreheater.

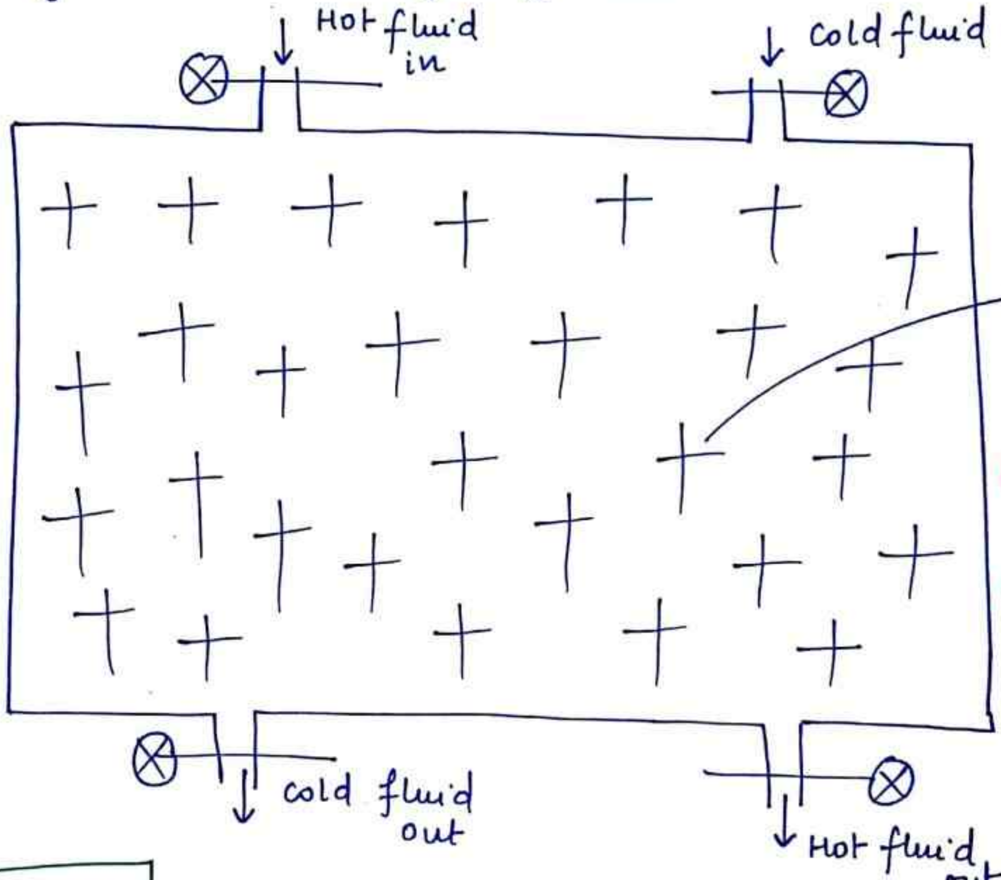
(2) In D.C.T.H.E., both hot and cold fluids physically mix-up with each other and exchange Q heat between them.

Example :- (a) Cooling Tower.



(b) Jet Condenser.

(3) In Regenerative or storage type H.E.,



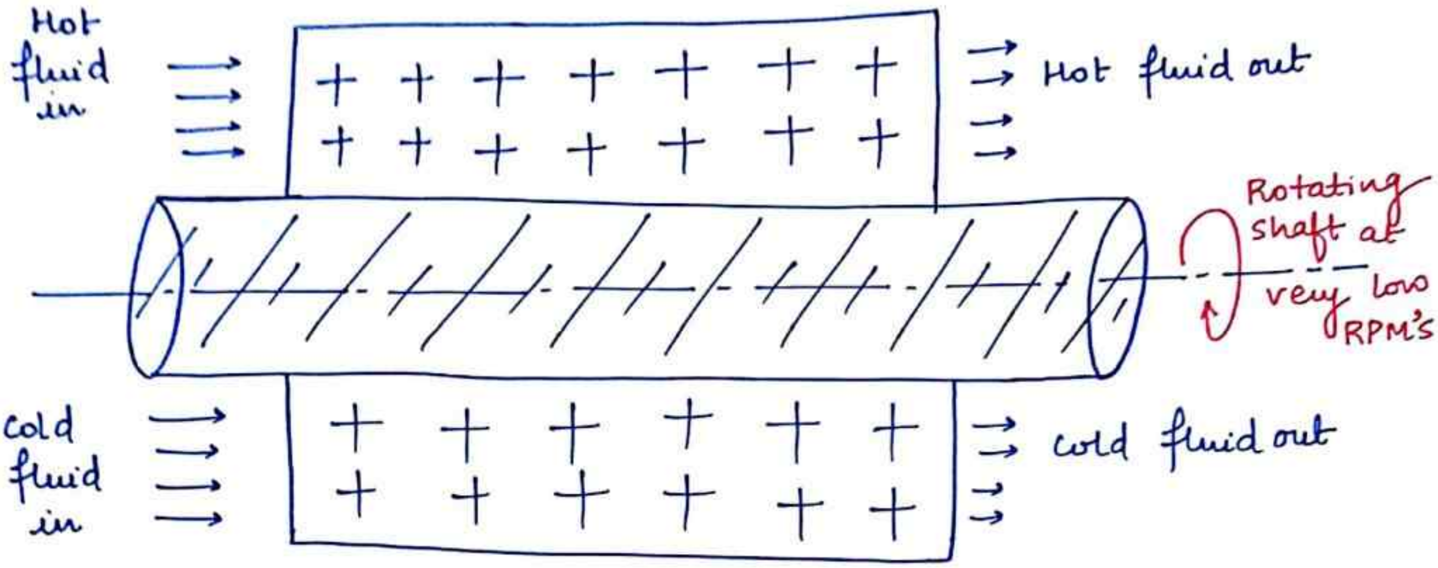
High heat capacity cellulose matrix (Hot & cold fluids alternatively flow through the H.E.)

Hot fluid heating the Matrix and cold fluid picking up heat from it.

Disadvantage :- No continuity of flows

Practical application :- Ljungstorm Air Preheater used in Gas Turbine Power Plants.

* Rotating Matrix Type Regenerative HE :-



Advantage :- ~~No need of stopping~~ continuity of fluid flows can be maintained

∴ No need of stopping and Restarting the fluid flows.

~~Disa~~

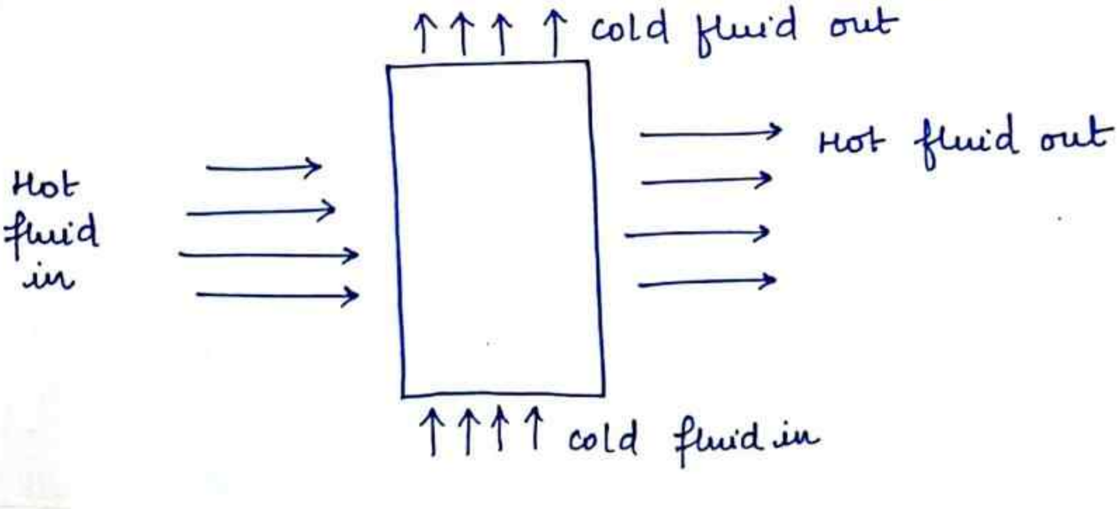
Disadvantage :- These may be some kind of fluid Mixing.

* Classification of Direct Transfer type HES :-

① Parallel flow HE (Hot & cold fluids travel in llel dirn)

② Counterflow HE (Hot & cold fluids travel in opposite dirn)

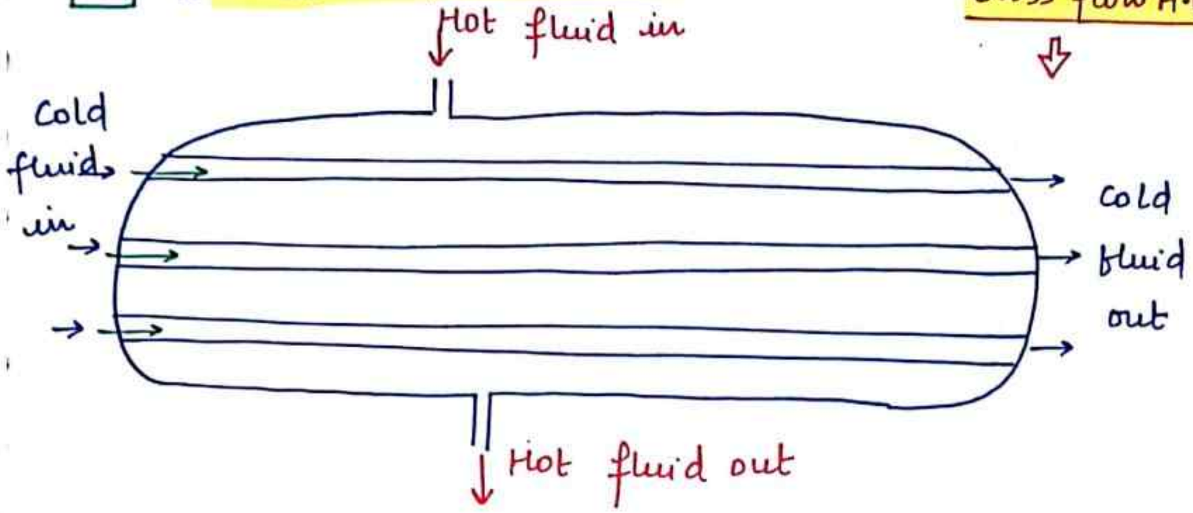
③ Cross flow HE (Hot and cold fluids travel in ⊥al dirn.) w.r.t. each other



Ex - ① shell and Tube H.E. :-

Cross flow H.E.

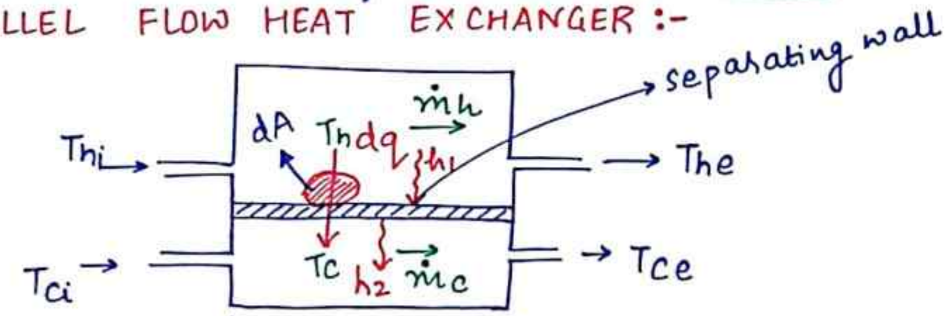
⇩



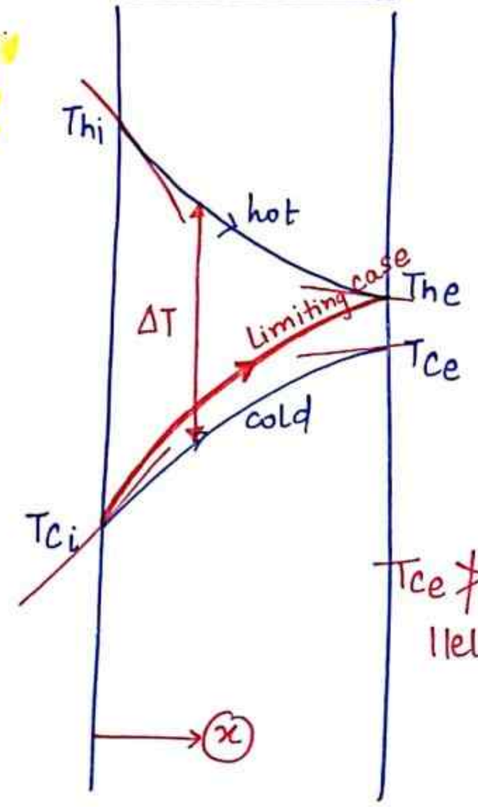
② Automobile Radiator.

* Temperature Profiles of Hot and Cold fluids :-

PARALLEL FLOW HEAT EXCHANGER :-



$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}}$$



x is the direction of Hot fluid flow.

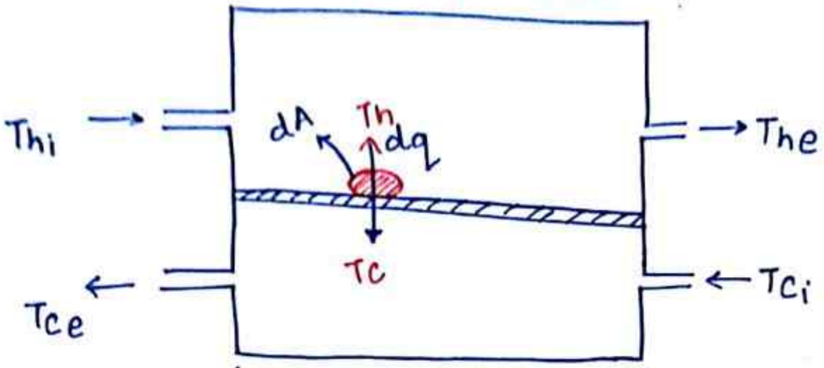
$\Delta T = (T_h - T_c)$

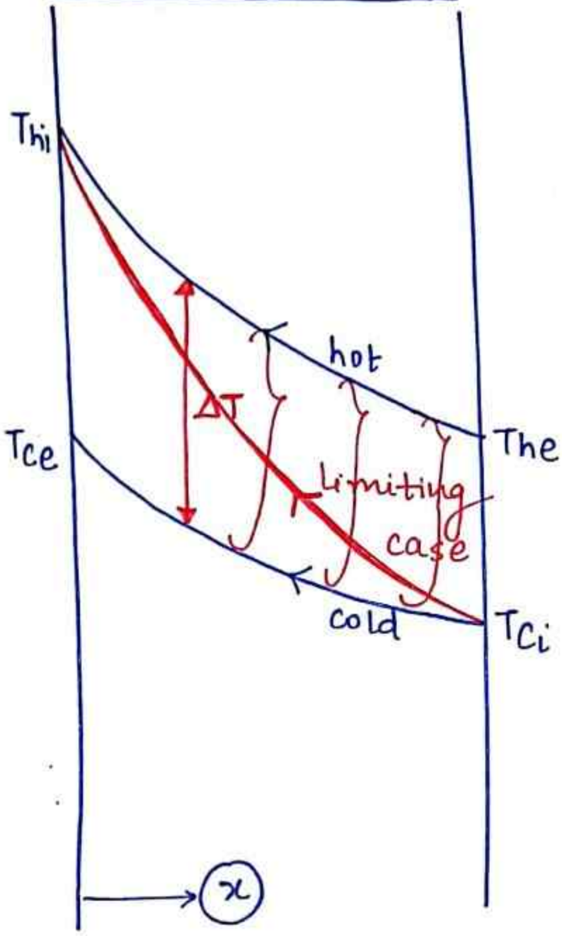
$= f(x)$

$dq = U \Delta T \, dA$

$T_{ce} \not> T_{he}$ in ||el flow H.E.

COUNTER FLOW :-





Tce can be greater than The only in counterflow H.E.

$\boxed{\text{NOTE}}$:-

The variation of ΔT with respect to 'x' is more pronounced in || flow H.E. as compared to that in counter flow H.E.

Hence, the irreversibility associated with heat transfer is higher in parallel flow H.E. as compared to that in counterflow H.E..

$$\left[(\Delta S)_{\text{T.D. universe in parallel flow H.E.}} > (\Delta S)_{\text{uni. in counter flow}} \right]$$

Thus counter flow H.E. is thermodynamically more efficient than parallel flow H.E. Hence for the same H.T. rate required in both the cases, counterflow heat Exchanger occupies lesser heat transfer area or more compact in size than parallel flow Heat Exchanger.

* M.T.D. (Mean Temperature Difference) :- (ΔT_m)

It is the parameter which takes into account the variation of ΔT with respect to x (direction of hot fluid flow) by averaging it all along the length of the Heat Exchanger from inlet to exit. and hence is defined from the equation

$$\boxed{Q = UA\,\Delta T_m} \quad (1)$$, where

Q = Total H.T. rate between Hot and cold fluids in entire H.E.

U = overall H.T. coefficient

A = Total H.T. area of the H.E.

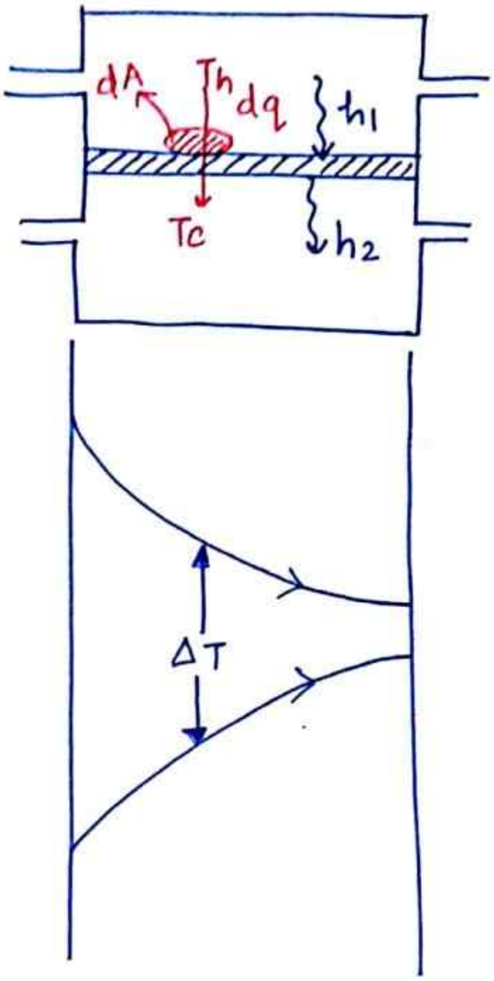
ΔT_m = MTD

$$\underset{(MTD)}{\Delta T_m} = \frac{1}{A}\int_{Inlet}^{Exit} \Delta T \, dA$$

← on comparing (1) and (2).

$$dq = U \Delta T \, dA$$

$$\int_{Inlet}^{Exit} dq = \int_{Inlet}^{Exit} U \, \Delta T \, dA$$

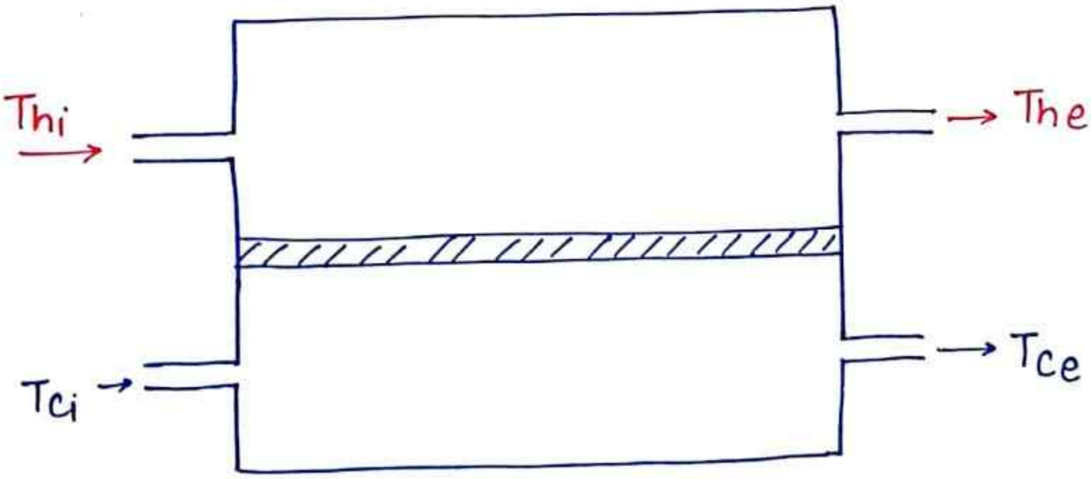


$$\Rightarrow \boxed{Q = U \int_{Inlet}^{Exit} \Delta T \, dA} \rightarrow (2)$$

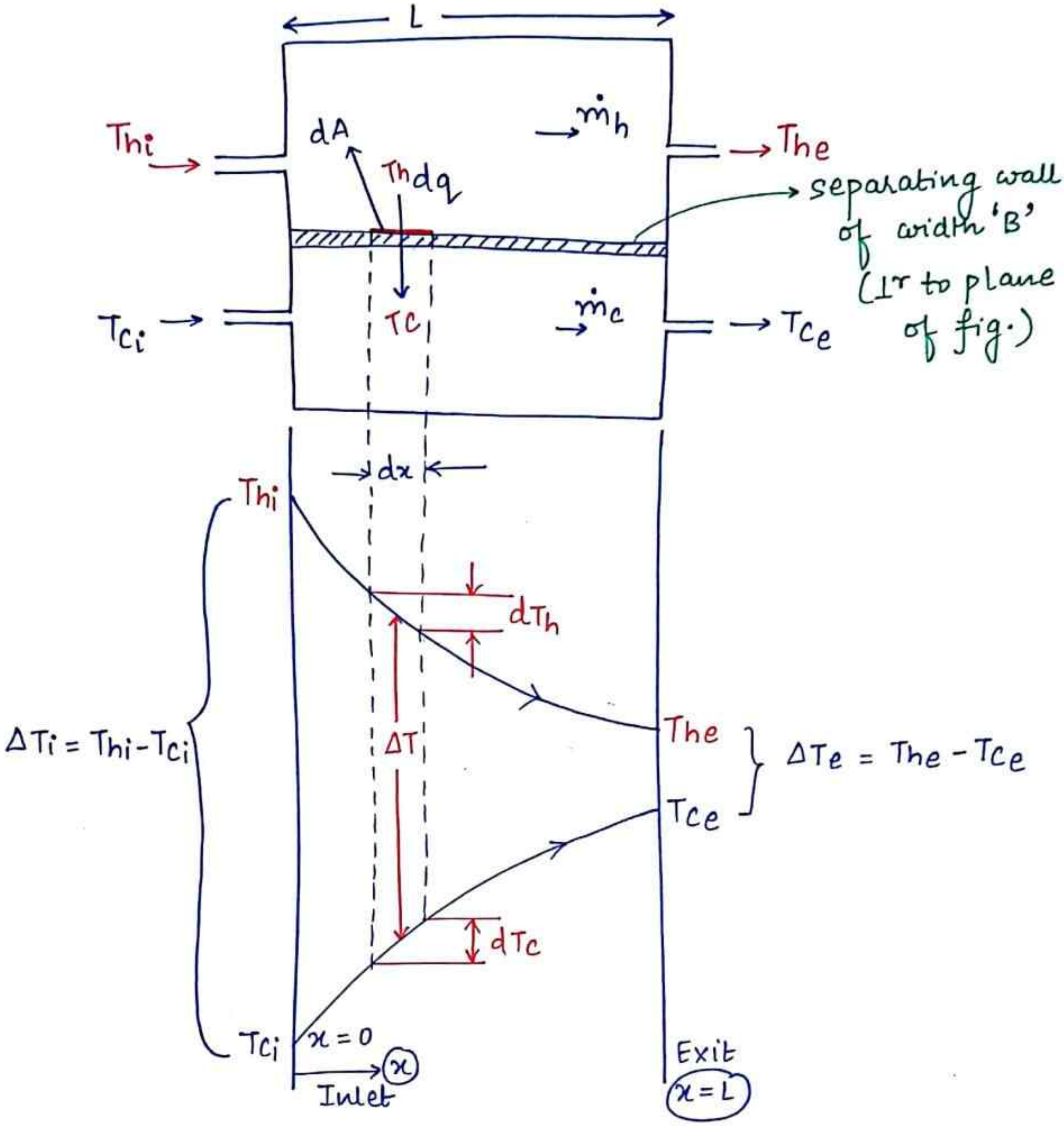
Comparing equ. (1) & equ (2)

$$\boxed{\underset{(MTD)}{\Delta T_m} = \frac{1}{A} \int_{Inlet}^{Exit} \Delta T \, dA}$$

To Derive an equation for MTD of Parallel flow H.E :-



" N.P. "



Consider differential H.T. Area dA of the H.E. of length 'dx' through which the differential H.T. rate b/w Hot and cold fluids is dq. Then $dq = U\Delta T\, dA$.

where $dA = B\,dx$.

and $\Delta T = T_h - T_c = f(x)$

$$\left[\begin{array}{l} \text{At } x = 0 \text{ (i.e. Inlet)} \Rightarrow \Delta T = \Delta T_i = T_{hi} - T_{ci} \\ \text{At } x = L \text{ (i.e. exit)} \Rightarrow \Delta T = \Delta T_e = T_{he} - T_{ce} \end{array}\right.$$

Also

$$dq = -\dot{m}_h c_{p_h} dT$$

$$= +\dot{m}_c c_{p_c} dT$$

$$\Delta T = T_h - T_c$$

$$d(\Delta T) = dT_h - dT_c$$

$$d(\Delta T) = -\frac{dq}{\dot{m}_h c_{p_h}} - \frac{dq}{\dot{m}_c c_{p_c}}$$

$$d(\Delta T) = -dq\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right]$$

$$\Rightarrow d(\Delta T) = -U\Delta T\, B\, dx\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right]$$

Now separating variables:-

$$\therefore \int_{\Delta T_i}^{\Delta T_e} -\frac{d(\Delta T)}{\Delta T} = \int_{x=0}^{L} UB\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right] dx$$

$$\Rightarrow \ln\frac{\Delta T_i}{(\Delta T_e)} = UBL\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right]$$

But $BL = A =$ Total H.T. area of H.E.

$$\therefore \ln\frac{\Delta T_i}{\Delta T_e} = UA\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right]$$

Q = Total H.T. Rate for entire H.E. $= \dot{m}_h c_{p_h}(T_{hi} - T_{he})$

$$= \dot{m}_c c_{p_c}(T_{ce} - T_{ci})$$

$$\Rightarrow \ln \frac{\Delta T_i}{\Delta T_e} = UA \left[\frac{T_{hi} - T_{he}}{Q} + \frac{T_{ce} - T_{ci}}{Q} \right]$$

$$= \frac{UA}{Q} \left[\Delta T_i - \Delta T_e \right]$$

$$\Rightarrow \boxed{Q = UA \left[\frac{\Delta T_i - \Delta T_e}{\ln \left(\frac{\Delta T_i}{\Delta T_e} \right)} \right]}$$

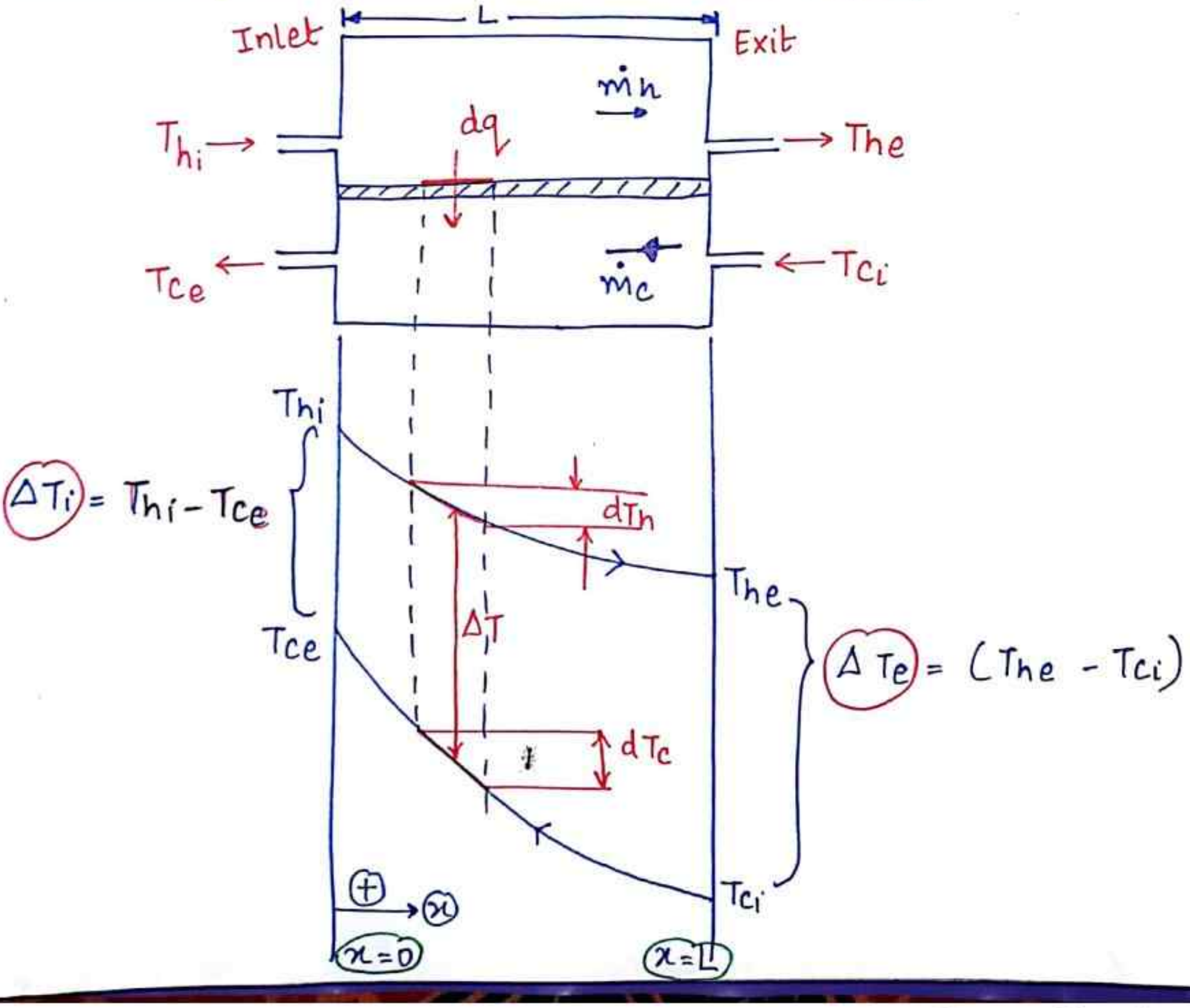
Comparing with

$$Q = UA \Delta T_m,$$

$$(\Delta T_m)_{||el} = \frac{\Delta T_i - \Delta T_e}{\ln \left(\frac{\Delta T_i}{\Delta T_e} \right)}$$

$$= (L.M.T.D.)_{||el\ flow}$$

* **LMTD of counter flow Heat Exchanger** :-



Scanned by CamScanner

$$dq = U \Delta T \, B dx$$

$$dq = -\dot{m}_h C_{ph} dT_h$$

$$= -\dot{m}_c C_{pc} dT_c$$

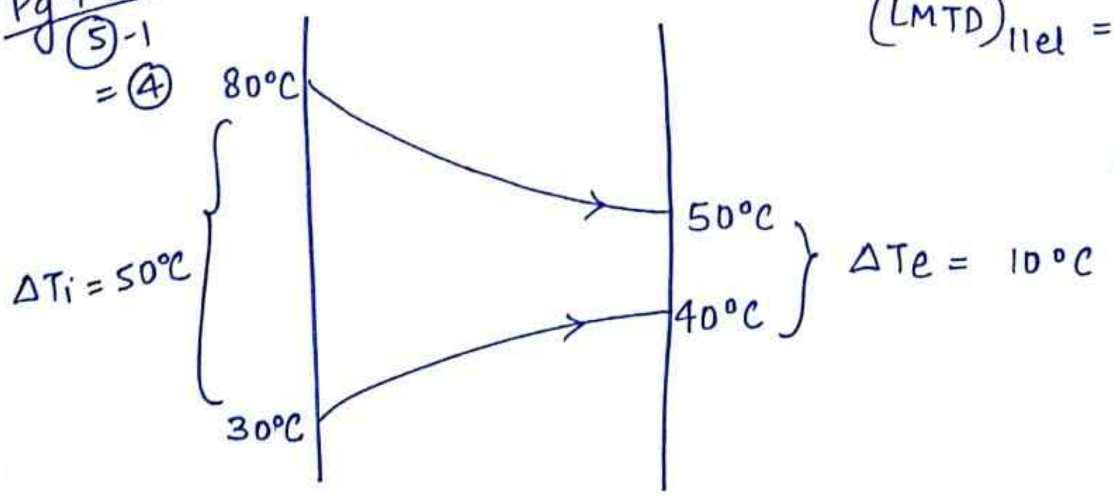
$$\boxed{(LMTD)_{counter} = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}}$$

NOTE :- 1. Even though the formulae for LMTD is the same for both parallel flow and counterflow Heat Exchanger. The definitions of ΔT_i and ΔT_e are different between them.

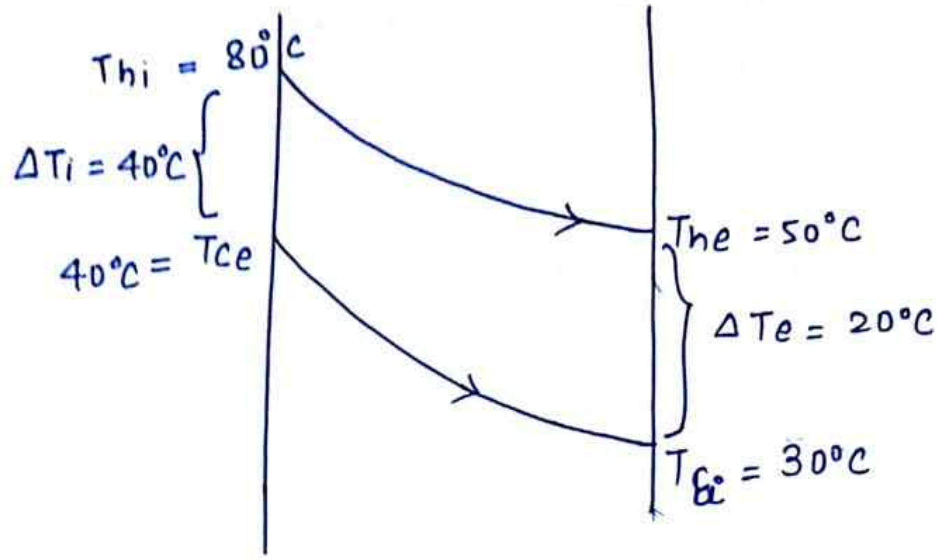
***

2. For the same inlet and exit temperatures of both hot and cold fluids employed/used in parallel flow and counterflow Heat Exchangers, the LMTD of counterflow H.E. is greater than LMTD of parallel flow H.E. This is the reason why counterflow H.E. is more compact in size than parallel flow H.E. for the same Heat transfer rate required in both the cases.

WB Pg 74 5-1 = 4

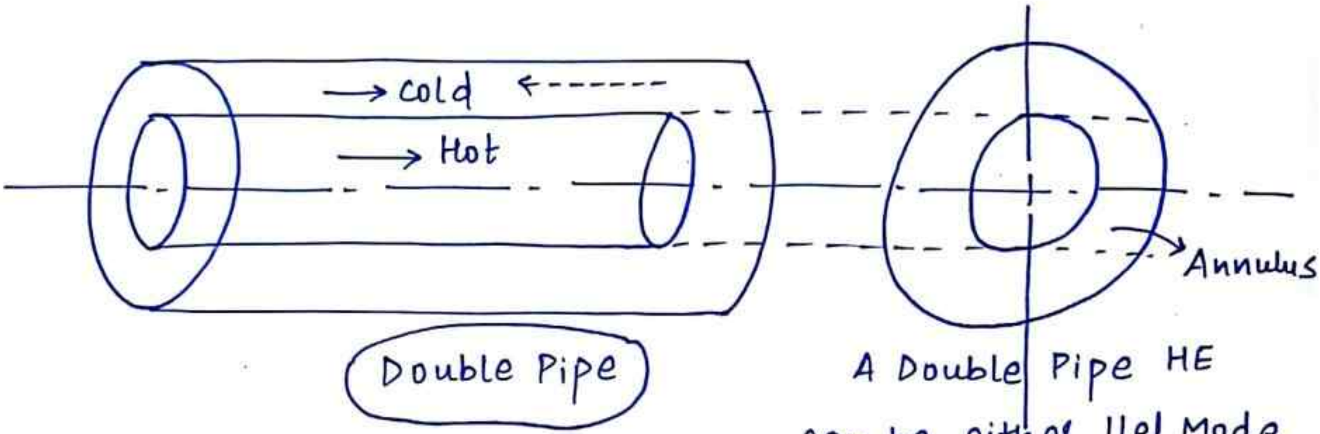


$$(LMTD)_{||el} = \frac{50-10}{\ln \frac{50}{10}} = 24.85°C$$



$$(LMTD)_{counter} = \frac{40-20}{\ln \frac{40}{20}}$$

$$= 28.85°C$$



A Double Pipe HE can be either llel Mode (or) counterflow Mode.
Crossflow is never possible here.

$$(LMTD)_{\substack{crossflow\\HE}} = (LMTD)_{\substack{counter\\flow}} \times F$$

where F = correction Factor

$$F < 1$$

Here $F = 26/28.85$

$$\boxed{F = 0.9}$$

(15) $T_{ci} = 10°C$ $\quad T_{hi} = 46°C$ $\quad \dot{m}_h = 25 \, l/s$

$T_{ce} = 38°C$ $\quad T_{he} = ?$

$\dot{m}_c = 19 \, lit/s$

$\rho = 1000 \, kg/m^3$ $\quad Cp_w = 4186 \, J/Kg \, K$

$$\dot{m}_h \, Cp_h (T_{hi} - T_{he}) = \dot{m}_c \, Cp_c \, (T_{ce} - T_{ci})$$

$$25 \, \cancel{25} \, (46 - T_{he}) = 19 \, (38 - 10)$$

SIR

$Cp_{hot \, water} = Cp_{cold \, water}$

$\rho_{hot \, water} = \rho_{cold \, water}$

$\dot{m} = \rho \times$ volume flow Rate

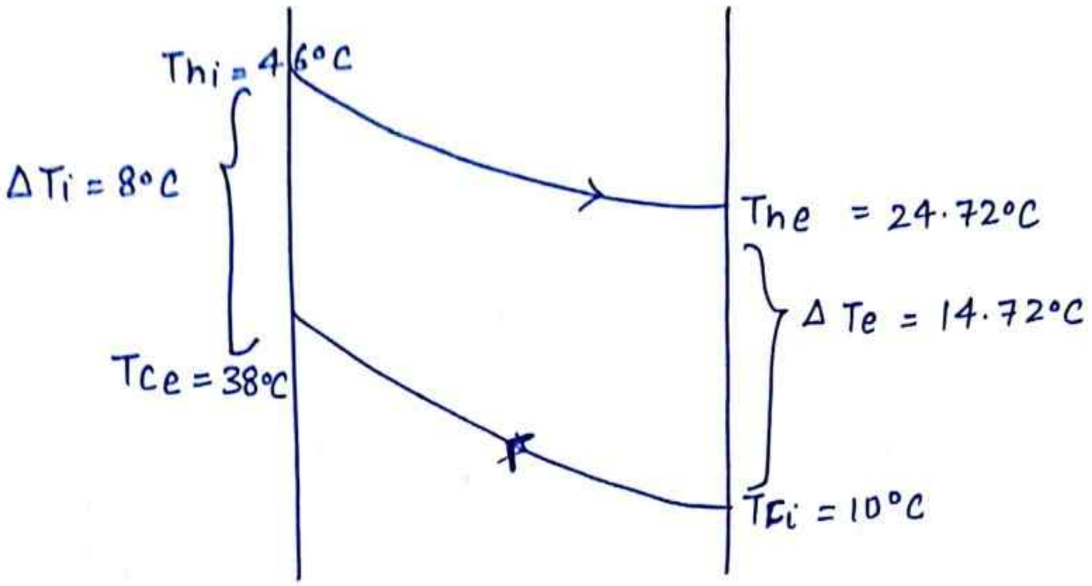
$\Rightarrow \dot{m} \propto$ volume flow Rate

Energy Balance Eqn :-

$$\dot{m}_h \, \cancel{Cp_h} (T_{hi} - T_{he}) = \dot{m}_c \, \cancel{Cp_c} (T_{ce} - T_{ci})$$

$$\Rightarrow 25 \, (46 - T_{he}) = 19 \, (38 - 10)$$

$$\boxed{T_{he} = 24.72°C}$$



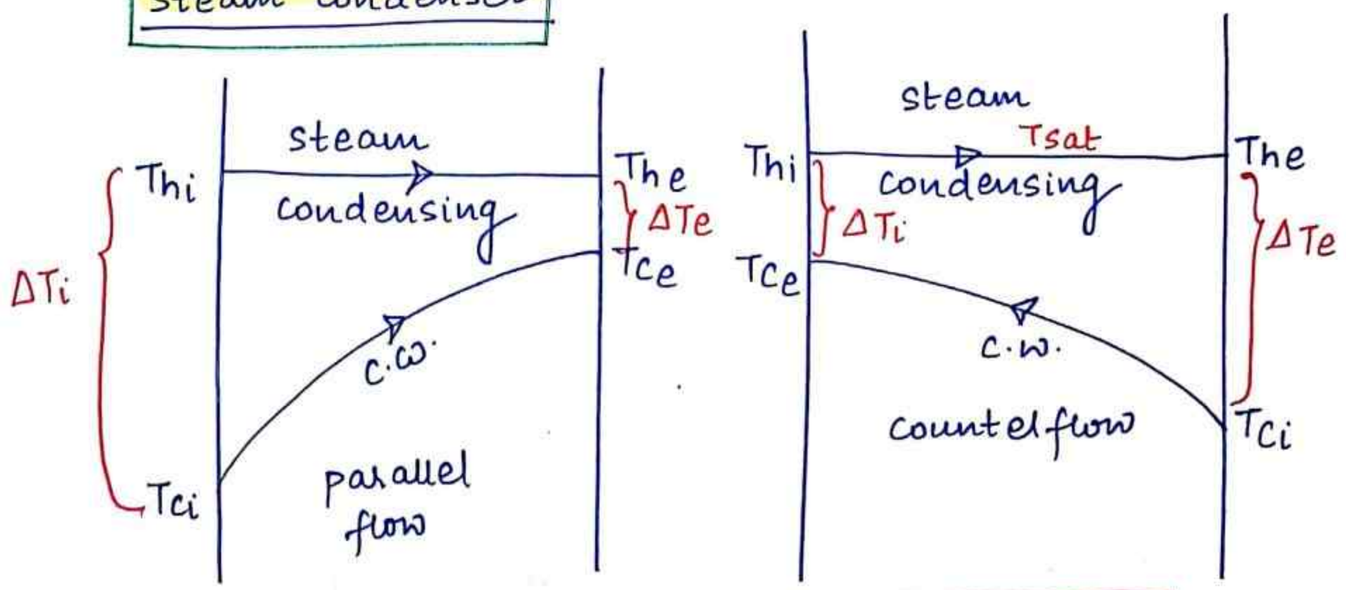
$$(LMTD)_{counterflow} = \frac{8-14.72}{\ln\left(\frac{8}{14.72}\right)}$$

$$= 11.02\,°C$$

* **Two special cases :-**

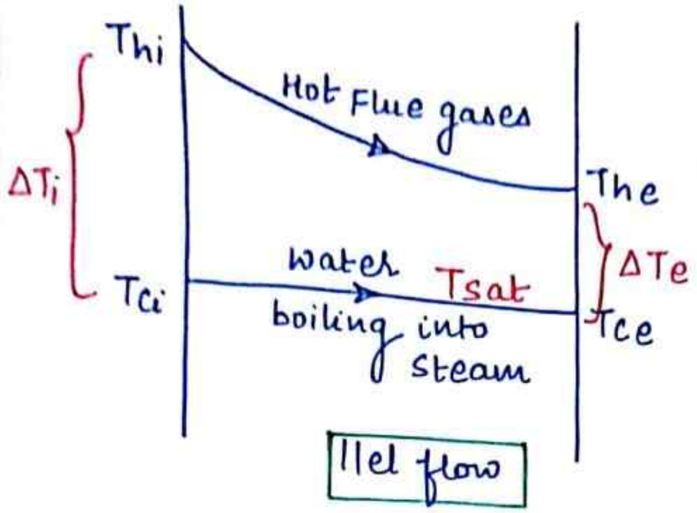
Case (I) :- when one of the fluids in the H.E. is undergoing phase change like a steam condenser (or) evaporator (or) steam generator.

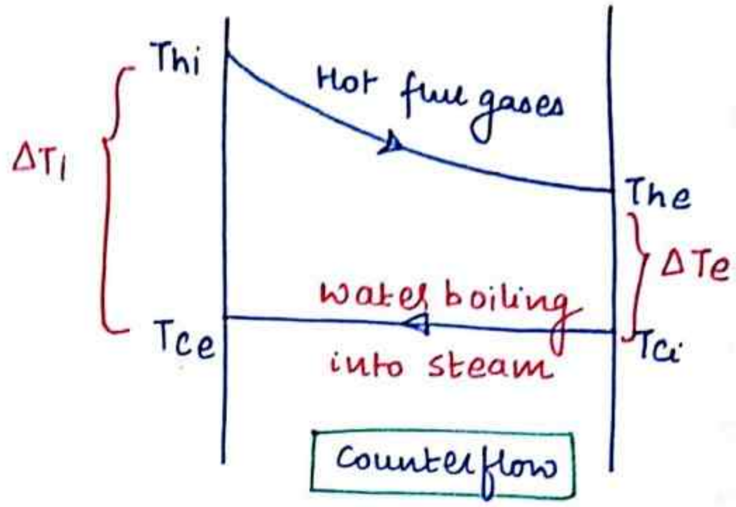
Steam Condenser



$$\boxed{(LMTD)_{||el} = (LMTD)_{counterflow}}$$

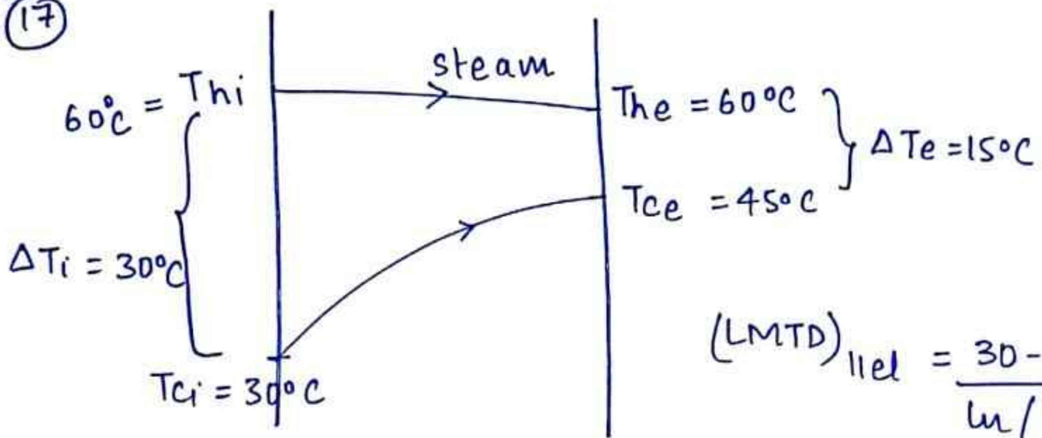
Steam Generator





$$\boxed{(LMTD)_{llel} = (LMTD)_{counter}}$$

WB
(17)



$60^\circ C$

$T_{ci} = 30$

$T_{ce} = 45$

$100 \longrightarrow 60$

$$(LMTD)_{llel} = \frac{30-15}{\ln\left(\frac{30}{15}\right)} = (LMTD)_{counter}$$

$$= 21.6^\circ C$$

~~Q.> When~~

$\boxed{\text{CASE II}}$:- When both Hot and cold fluids have equal capacity rates in a counter flow heat exchanger that is when

$\dot{m}_h C_{Ph} = \dot{m}_c C_{Pc}$ in a counterflow HE,

$(\dot{m} C_P)$ product is called heat capacity Rate.

Then from Energy-balance Eqn.,

$$\dot{m}_h \cancel{c_{p_h}} (T_{hi} - T_{he}) = \dot{m}_c \cancel{c_{p_c}} (T_{ce} - T_{ci})$$

$$\Rightarrow (T_{hi} - T_{he}) = (T_{ce} - T_{ci})$$

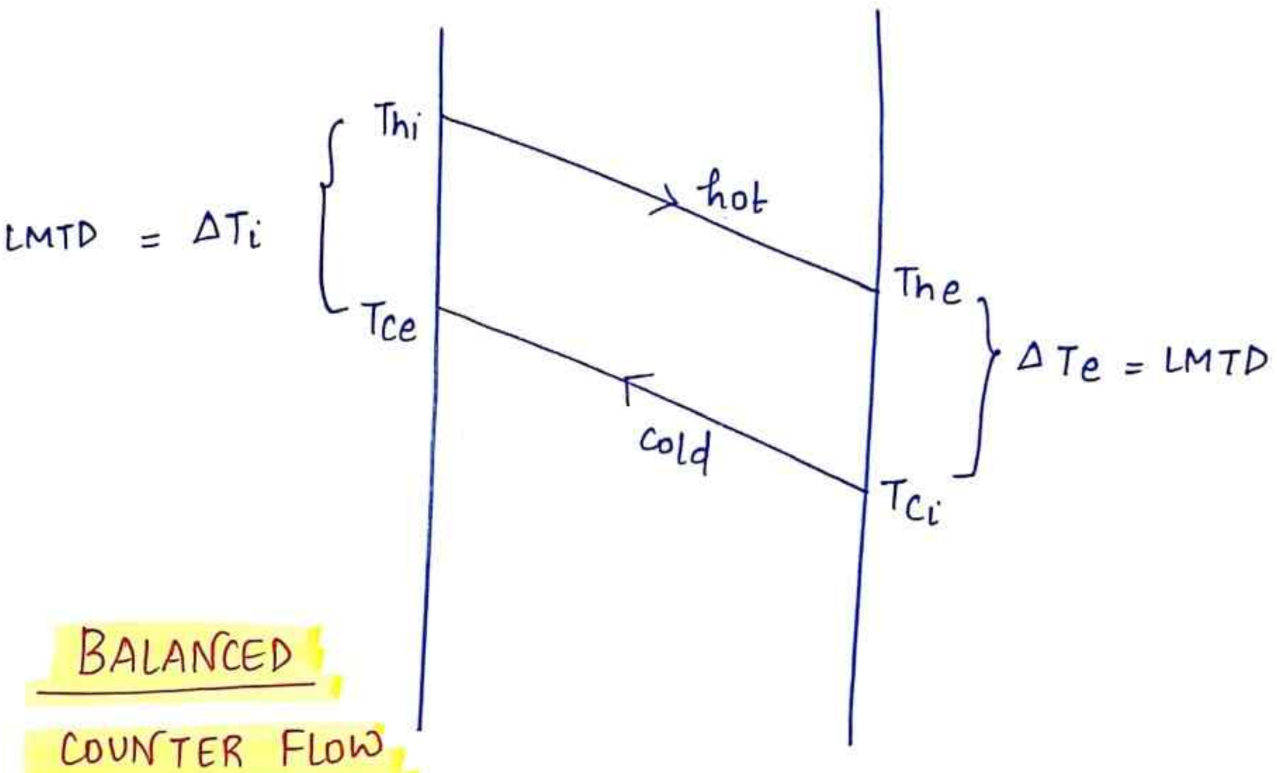
$$\Rightarrow (T_{hi} - T_{ce}) = (T_{he} - T_{ci})$$

$$\Delta T_i = \Delta T_e$$

Then $(LMTD)_{counterflow} = \dfrac{\Delta T_i - \Delta T_e}{\ln\left(\dfrac{\Delta T_i}{\Delta T_e}\right)} = \dfrac{0}{0}$ (undefined)

Then From L'hospital's Rule,

$(LMTD)_{counterflow}$ = Either ΔT_i (OR) ΔT_e



BALANCED

COUNTER FLOW

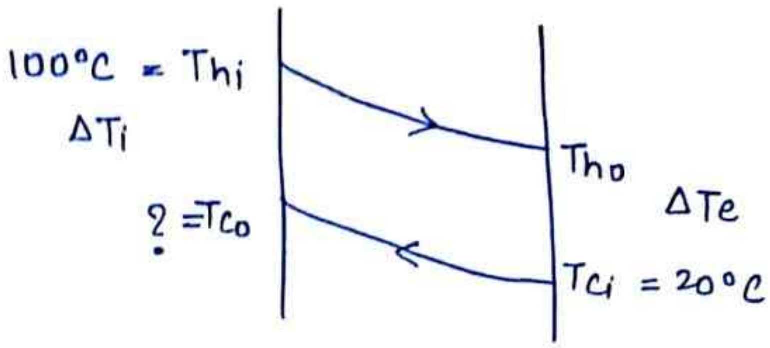
H.E.

⑤

LMTD = 20°C

$\dot{m}_c = 2\dot{m}_h$

$C_{p_h} = 2 C_{p_c}$



$$\cancel{e}\,20 = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

$$\cancel{e}\ 20 = \frac{(100 - T_{co}) - (T_{ho} - 20°)}{\ln\left(\frac{100 - T_{co}}{T_{ho} - 20}\right)}$$

$$\dot{m}_c C_{p_c} \Delta T_i = \dot{m}_h C_{p_c} \Delta T_e$$

$$\cancel{2}\cancel{\dot{m}_h}\,\cancel{C_{p_c}}(100 - T_{co}) = \cancel{\dot{m}_h}\,\cancel{2}\cancel{C_{p_c}}(T_{ho} - 20°)$$

$$T_{co} - T_{ho} = -20 - 100$$

$$T_{co} - 120 + T_{co} = -120$$

$$-T_{co} + T_{ho} = (120)$$

$$T_{ho} = 120 + T_{co}$$

$$T_{ho} - T_{co} = 120$$

$$\ln\left(\frac{100 - T_{co}}{T_{ho} - 20}\right) = -T_{ho} - T_{co} + 80 - 20$$

$$= -(T_{ho} + T_{co}) + 60$$

Scanned by CamScanner

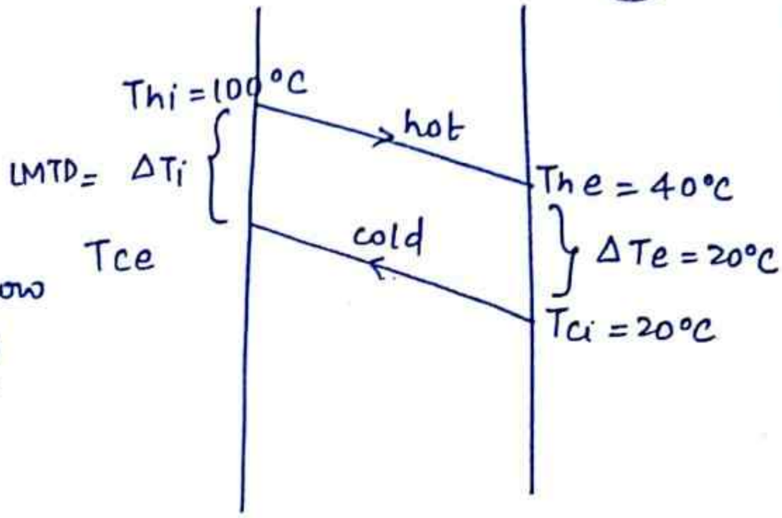
SIR $\dot{m}_h = \frac{1}{2}\dot{m}_c$

$cp_h = 2\,cp_c$

$\Rightarrow \dot{m}_h cp_h = \dot{m}_c cp_c$

and HE is counterflow

(Balanced)



$LMTD = 20° = \Delta T_i = T_{hi} - T_{ce} =$

$100 - T_{ce}$

$T_{ce} = 100 - 20 = 80°C$

Q3 SIR $\dot{m}_h cp_h = \dot{m}_c cp_c$

and HE is counterflow (balanced)

$\therefore \; LMTD = T_{hi} - T_{ce}$

$= 30°C$

Q6 $cp_h = 2$ $\qquad cp_c = 4$

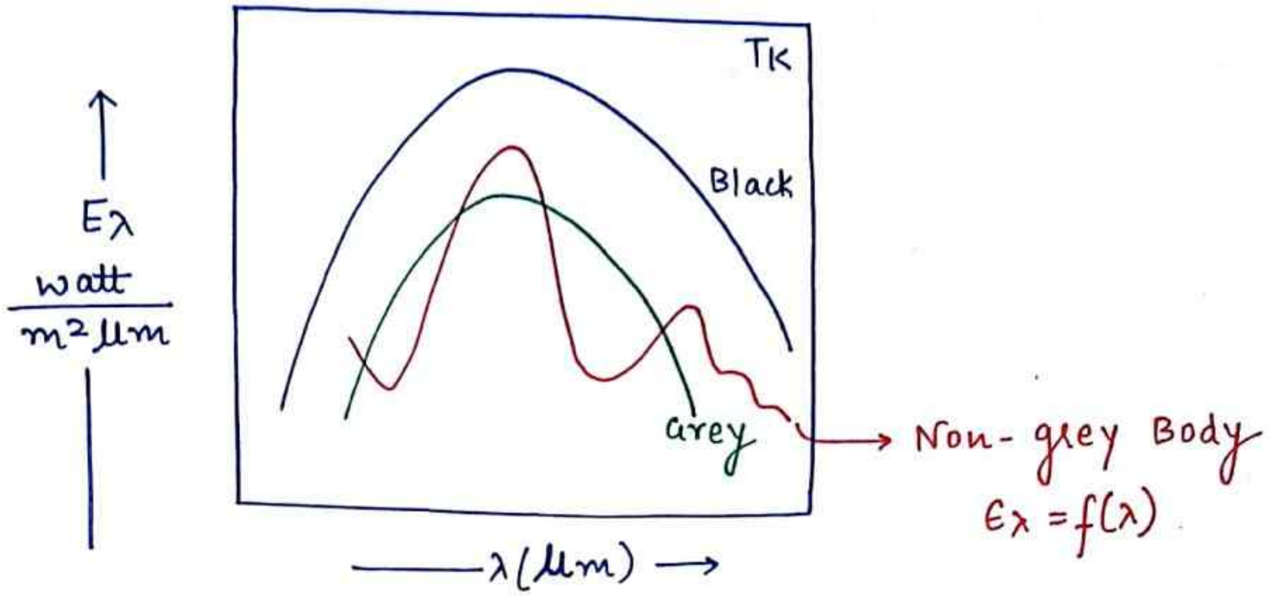
$m_h = 5$ $\qquad m_c = 10$

$T_{hi} = 150°C$ $\qquad T_{ci} = 20°C$

$T_{ho} = 100°C$

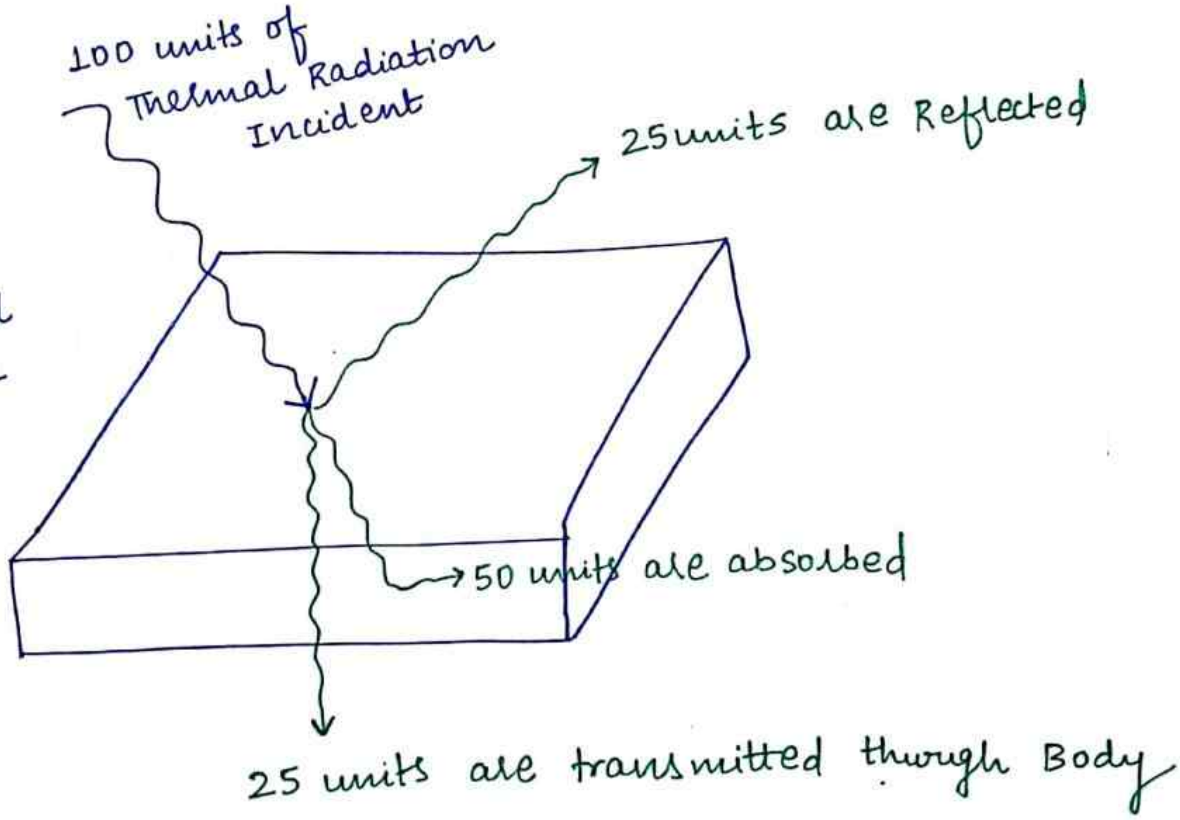
$\dot{m}_h cp_h (150° - 100°) = \dot{m}_c cp_c (T_{ce} - 20°C)$

$T_{ce} = 32.5°C$



* $\boxed{\text{Absorptivity } (\alpha), \text{Reflectivity } (\rho) \text{ and Transmissivity } (\tau)}$:-

For Black body, which absorbs all thermal Radiation incident, $\alpha_b = 1$



All the above Radiation properties defined ~~as~~ change with wavelength of ~~instant~~ incident thermal Radiation, surface Roughness of the body and the fixed Tempr. $\longrightarrow$

Absorptivity $(\alpha) = \frac{50}{100} = 0.5$ = fraction of Radiation energy incident upon a surface which is absorbed by it.

Reflectivity $(\rho) = \frac{25}{100} = 0.25$ = fraction of Radiation energy incident upon a surface which is Reflected by it.

Transmissivity $(\tau) = \frac{25}{100} = 0.25$ = fraction of Radiation energy incident upon a surface which is transmitted through it.

∴ For any surface, $\alpha + \rho + \tau = 1$

✓ for opaque surface, which does not Transmit any energy,

$\tau = 0$

✓ ∴ for opaque surface, $\alpha + \rho = 1$.

✓ for black body, which absorbs all Thermal Radiation Incident, $\alpha_b = 1$

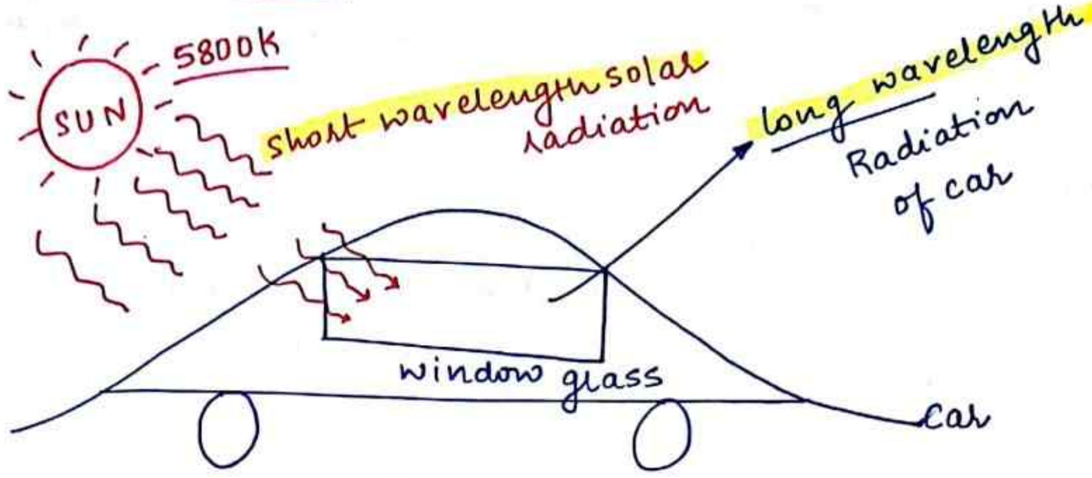
✓ Metals have high Reflectivity (ρ) as compared to Non-metals.

$\boxed{\text{NOTE}}$:- Metal surfaces are more reflecting to the thermal Radiations. This is the reason why metallic surfaces are made of Cu (or) Al are generally used as radiation shields in the furnaces to reduce radiation heat Exchange.

✓ Gases like O_2, N_2 etc have high Transmissivity (τ) i.e. Transparent to thermal Radiation.

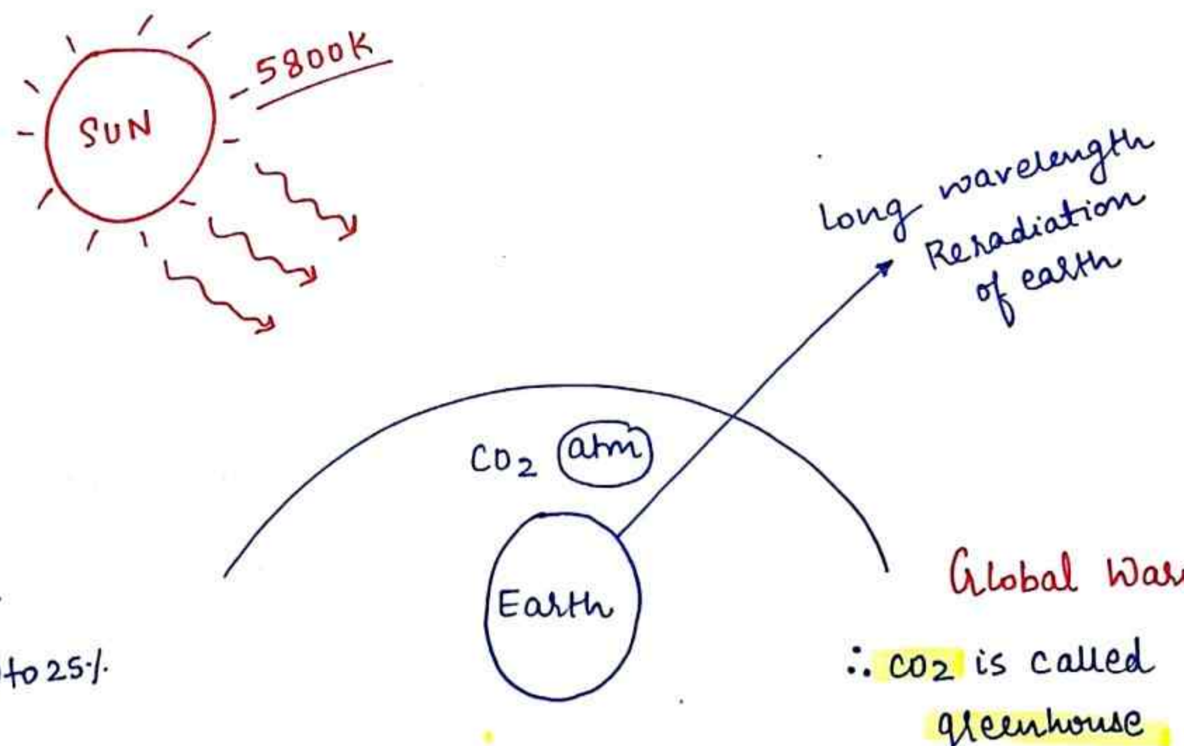
* Practical Example :-

①



The window glass of a car is very much transparent to the short wavelength solar radiation falling upon it. But the same window glass almost becomes opaque to the long wavelength re-radiation given by inside of the car. thus trapping energy inside the car hence increasing its temperature.

②



Desert

↓

$\phi = 20$ to 25%

(R.H.)

Global Warming

∴ CO_2 is called greenhouse gas.

H_2O (w.v.) is also greenhouse gas.

as the surface roughness of a Body decreases by polishing it, the reflectivity of the surface will increase. (125)

Hence, Eg:- highly polished 'Al' or 'Cu' shields having very good reflectivity are generally used in the furnaces to reduce radiation heat exchange.

* LAWS OF THERMAL RADIATION :-

① Kirchoff's Law of Thermal Radiation :- The law states that whenever a Body is in thermal Eqm. with its surroundings, its emissivity is equal to its absorptivity.

$$\boxed{\epsilon = \alpha}$$

valid for Black nor Black. for eqm. Ther or non-eqm. Thermal

A good absorber is always a good emitter.

Ex :- for a black body, $\alpha_b = 1$
$\epsilon_b = 1$.

② Planck's Law of Thermal Radiation :-

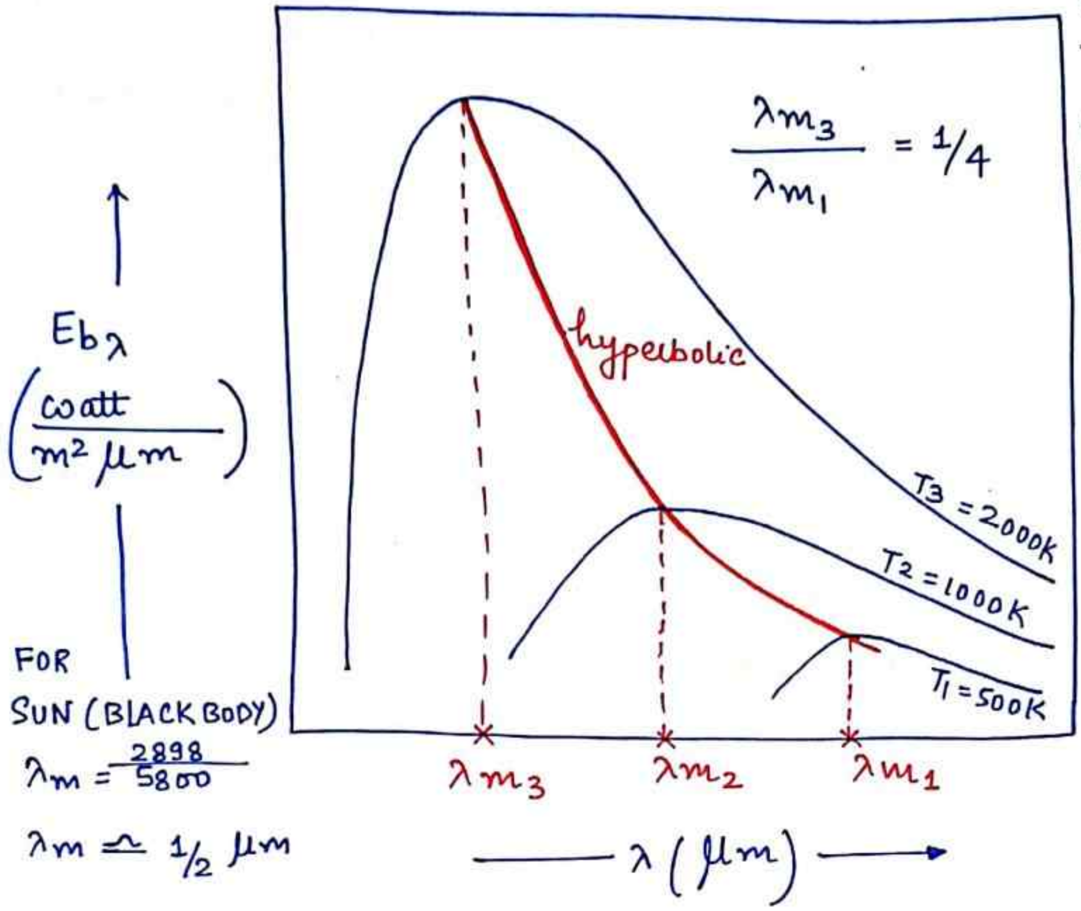
$$E_{b\lambda} = f(\lambda, T) \quad T \text{ in Kelvin}$$

The law states that the monochromatic emissive power of a Black body is dependent on both absolute tempr. of Black Body and also on wavelength of ~~ene~~ Radiation energy emitted 'λ'.

$$E_{b\lambda} = \frac{2\pi C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1 \right)} \quad \text{watt}/m^2\text{-}\mu m$$

C_1 and C_2 are experimental constants.

The above functional relationship among the 3 variables can be graphically represented as :-



The ratio between the area under Top-most curve on x-axis and the area under Bottom most curve on x-axis will be equal to $4^4 = \boxed{256}$.

FOR SUN (BLACK BODY)

$\lambda m = \frac{2898}{5800}$

$\lambda m \simeq 1/2\ \mu m$

λm = wavelength at which Eb_λ is maximum at a given absolute Temperature of Black Body.

At a given absolute temperature of a Black Body as wavelength 'λ' increases, Eb_λ also increase reaches a maximum and then decreases.

Also as absolute tempr. of Black Body increases (each time getting doubled). Eb_λ value enormously increases but now most of thermal radiation at higher ~~the~~ tempr.'s will be shifted to shorter wavelengths.

As T increases $\Rightarrow$ λm decreases.

i.e. $\lambda m \propto \frac{1}{T}$

i.e. $\boxed{\lambda m\, T = \text{a constant} = 2898\ \mu m\text{-}K}$ This eqn. is called Wein's displacement law.

$\hookrightarrow$ ③

wein's displacement law

Optical pyrometer to measure very high Temps. uses

$\boxed{\lambda_m T = c}$

④ **STEFAN - BOLTZMAN' Law** :- The law states that the Total hemispherical emissive power of a Black body is directly proportional to the fourth power of the absolute temperature of the black Body.

$$E_b \propto T^4 \text{ (T in K only)}$$

$$\Rightarrow E_b = \sigma T^4 \text{ watt/m}^2$$

σ = stefan - Boltzman' constant

$= 5.67 \times 10^{-8}\ W/m^2K^4$

$$\Rightarrow E_b = \int_0^\infty E_{b\lambda}\, d\lambda = \int_0^\infty \frac{2\pi c_1}{\lambda^5\left(e^{c_2/\lambda T} - 1\right)} d\lambda$$

$$= \sigma T^4 \ \omega/m^2$$

Planck's law → Stefan - Boltzman' Law

Planck's law → Wein's displacement Law

For a Non-Black Body, whose emissivity is ϵ, The Total hemispherical emissive power of Non-Black Body $= E = \epsilon E_b \ \frac{watt}{m^2}$

$$E = \epsilon \sigma T^4 \ W/m^2$$

If 'A' is the Total surface area of Non-Black body,

The Total Radiation energy emitted from entire Non-Black Body $= EA$ watt

$= \epsilon \sigma T^4 A$ watt

Pg. 80

(29) +1

= (30) $E_1 = 500\ W/m^2$ —— T_1 $\qquad \epsilon = C$

$E_2 = 1200\ W/m^2$ —— T_2 $\qquad \frac{T_1}{T_2}$

SIR $\quad E_1 = \epsilon_1 \sigma T_1^4 = 500\ W/m^2, \quad E_2 = \epsilon_2 \sigma T_2^4 = 1200\ W/m^2$

But $\epsilon_1 = \epsilon_2$

$$\Rightarrow \left(\frac{T_1}{T_2}\right)^4 = \frac{500}{1200}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(5/12\right)^4 \quad = 0.803$$

(32) Sun is a black body

From wein's displacement law,

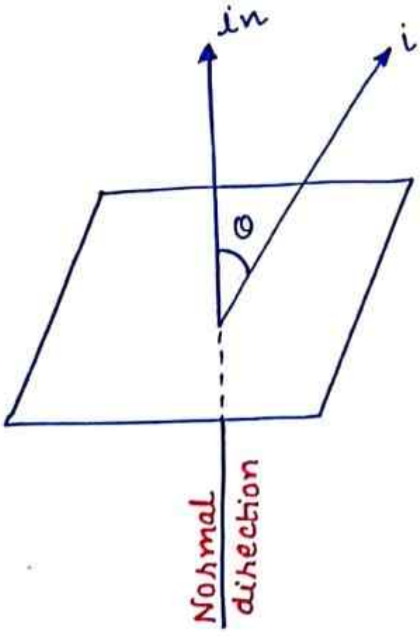
$$\lambda m_1 T_1 = \lambda m_2 T_2$$

$$5800 \times 0.50 = \lambda m_2 \times 1000$$

$$\lambda m_2 = 2.90\ \mu m$$

22/09/2016

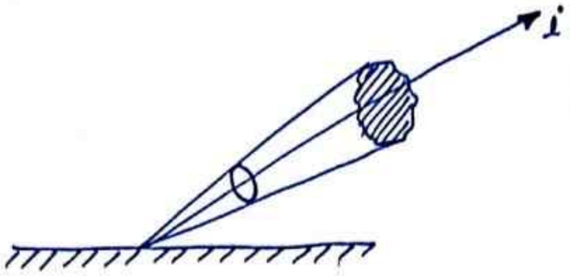
(5) LAMBERT'S COSINE LAW :-



$\boxed{i = i_n \cos\theta}$

i_n = Normal Intensity of Radiation.

i = intensity of Radiation along any direction making an angle 'θ' wrt Normal direction



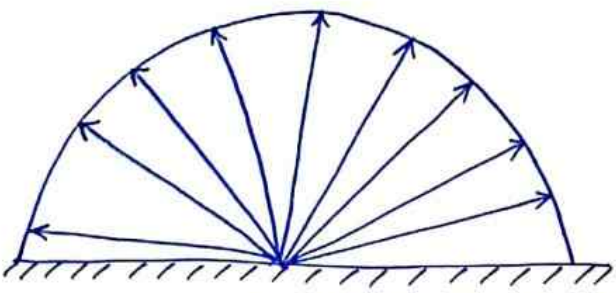
$$\frac{\text{Joule}}{\sec m^2\text{-steradian}} = \frac{\text{watt}}{m^2\text{-steradian}}$$

Steradian is the unit of solid angle.

$\boxed{i}$ → Intensity of Radiation 'i' along a given specified dirn. is defined as the radiation energy emitted from the surface of the body per unit time per unit area normal to that direction and per unit solid angle about that dirn.

$$i = \frac{dE}{d\omega} \quad \text{watt}/m^2\text{- steradian}.$$

Total hemispherical emissive power $= E = \int i \, d\omega$ watt/m^2.



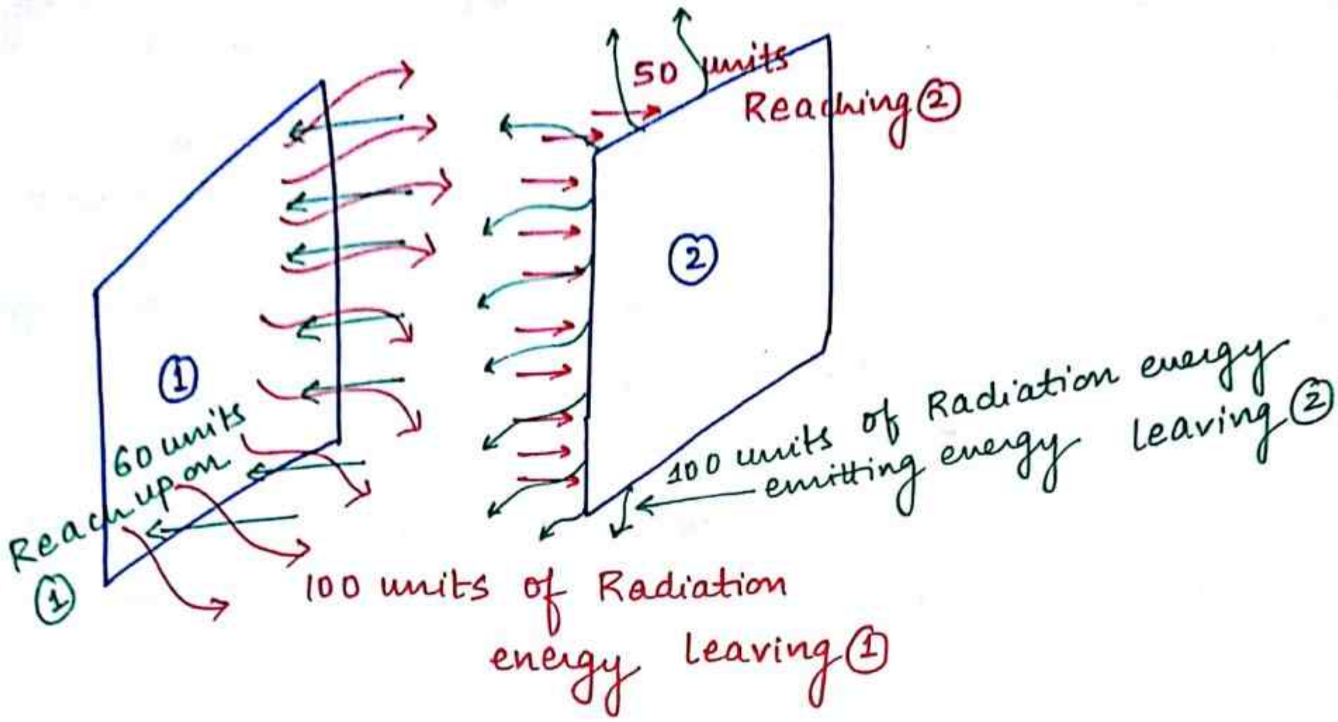
A Diffuse surface has the same intensity of Radiation along all the directions.

i.e. for Diffuse surface, i is independent of Direction.

Ex:- Blackbody is a diffuse surface.

∴ for Black body, $E_b = \pi i_n \, \omega/m^2$.

* **SHAPE FACTOR (OR) VIEW FACTOR (OR) CONFIGURATION FACTOR :-**



$F_{12} = \dfrac{50}{100} = 0.5$ = Fraction of Radiation energy leaving surface ① that reaches surface ②

$F_{21} = \dfrac{60}{100} = 0.6$ = Fraction of Radiation energy leaving surface ② that reaches surface ①.

In General,

F_{mn} = Fraction of Radiation energy leaving surface 'm' that reaches surface 'n'.

$$\boxed{0 \le F_{mn} \le 1}$$

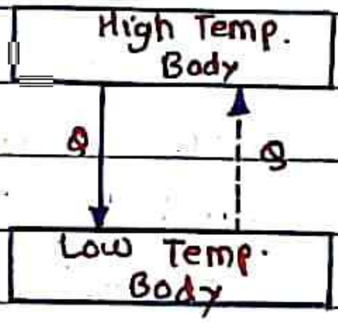
NOTE - The shape Factor b/w 2 surfaces / bodies is independent of their temperatures, their emissivities, but depends only on how the 2 surfaces are geometrically oriented with respect to each other. (Also their sizes & shapes can influence the shape factor).

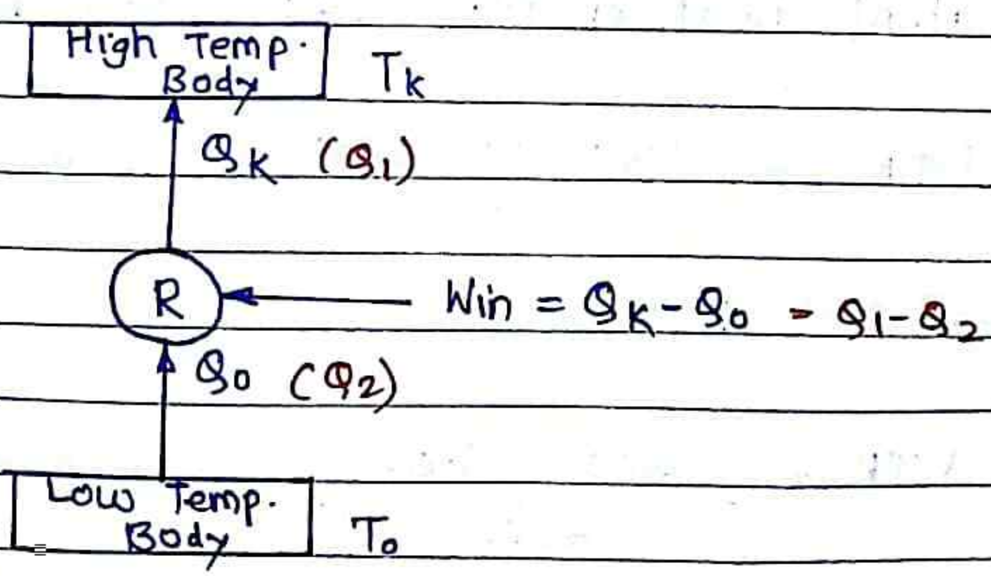
Scanned by CamScanner

Refrigeration → Refrigeration is the cooling of a system below the temperature of its surrounding. Refrigeration / heat pumps or work consuming plants.

Refrigeration is completely based on second law of thermodynamics.

Clausius Statement → It is impossible to construct a device which operating in a cycle, will produce no effect other than transfer of heat from a cooler to a hotter body. ie. Heat can not flow of itself from a body at a lower temperature to a body at a higher temperature. Some work must be expended to achieve this.





There must be some work input Win to transfer heat from low temp. body to high temp. body.

For in a single process it may be possible to obtain removal of heat from a low temp. body without doing ext. work eg. by the evaporation of a refrigerant for instance liquid Freon 22 after throattling expansion.

Unit of Refrigerating capacity →

The standard unit of refrigeration in vogue is ton of refrigeration or simply ton denoted by the symbol TR. It is equivalent to the production of cold at the rate at which heat is to be removed from one US tonne of water at 32°F ie. 0°c to freeze it to ice at 32°F (0°c) in one day or 24 hrs.

1 US tonne = 2000 lb

Latent heat of ice 144 BTU/lb

$$1\,TR = \frac{1 \times 2000 \times 144}{24} = 12000\ BTU/hr$$

But 1 BTU = 1.055 KJ

$$1\,TR = \frac{12000 \times 1.055}{3600} = 3.517\ KW$$

$$\boxed{1\,TR = 3.517\ KW}$$

In metric SI,

$$1\,TR = \frac{1000\ kg \times \text{Latent heat of ice}}{24 \times 3600}$$

$$= \frac{1000 \times 333.55}{24 \times 3600}$$

$$1\,TR = 3.86\ kW.$$

For US tonne 1 US tonne = 911 kg.

$$1\,TR = \frac{911 \times 333.55}{24 \times 3600} = 3.517\ KW$$

* Energy Ratios or Coefficient of performance

The performance of a heat engine is described by its thermal efficiency. The performance of refrigerating machine or a heat pump is expressed by the ratio of useful heat to work, called the energy ratio or coefficient of performance (COP).

$$(COP)_c = \frac{\text{Desired effect}}{\text{Work Input}} = \frac{\dot{Q}_0}{W_{net}} = \frac{Q_2}{W}$$

/ Cooling Energy Ratio.

$$= \frac{Q_2}{Q_1 - Q_2}$$

Heating energy ratio or cop for heating

$$(COP)_{Heating} = \frac{Q_R}{W_{net}} = \frac{Q_1}{W}$$

$$= \frac{Q_1}{Q_1 - Q_2}$$

$$\boxed{(COP)_{Heating} = (COP)_{cooling} + 1}$$

* For vapour compression systems $(COP)_{cooling}$ is of the order of 3 for water cooled and 2 for air cooled air conditioning applications & 1 for domestic refrigerators.

* For air cycle refrigeration systems $COP \approx 1$ & for vapour absorption systems it is well below unity.

* Power Consumption of a refrigerating machine → power consumption $\dot{W}$ of a refrigerating machine is determined in terms of kW. Power consumption of motors is rated in HP

$$HP = \frac{\dot{W}}{0.746} \quad \ldots \text{where } \dot{W} \text{ is in kW}$$

$$HP/kW \text{ refrigeration} = \frac{\dot{W}}{0.746\,\dot{Q}_0}$$

$$= \frac{4.71\,\dot{W}}{\dot{Q}_0}$$

$$\boxed{HP/TR = \frac{4.71}{COP}}$$

If the horsepower is metric

$$HP = \frac{\dot{W}}{0.736}$$

$$\boxed{HP/TR = \frac{4.78}{COP}}$$

* Heat pump vs electric resistance heater → Cop heat pump is always greater than unity.

$$(COP)_{HP} = \frac{Q_R}{W}$$

$$Q_R = (COP)_{HP} \times W$$

$$= (1 + (COP)_R) \times W$$

Even if $(COP)_R = 0$ Q_R will be equal to W. The value of $(COP)_{HP}$ for ACS in order of ~~4~~ 3. Hence Power consumption is using electric resistance heater

$$\dot{W} = \dot{Q}_R$$

$$(COP)_{HP} = \frac{\dot{Q}_R}{\dot{W}}$$

$$3 = \frac{\dot{Q}_R}{\dot{W}}$$

$$\dot{W} = \frac{1}{3} \dot{Q}_R$$

Thus power consumption of the heat pump is very much lower.

* Energy efficiency ratio [EER] →

$$= \frac{\text{Cooling Capacity in watts}}{\text{Input wattage in watts}}$$

* 3 star 1·5 TR split AC (I/P wattage 1800 watts)

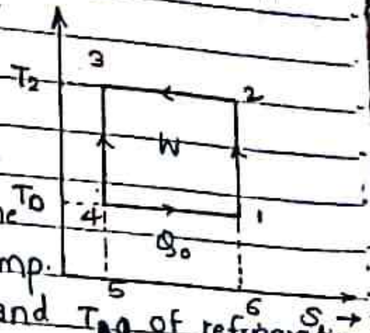
$$EER = \frac{1 \cdot 5 \times 3 \cdot 517}{1800} = 2 \cdot 93$$

* 3 Star 1·5 TR Window AC [I/P wattage 1950 watt]

$$EER = \frac{1 \cdot 5 \times 3517}{1950} = 2 \cdot 70$$

Reversed Carnot Cycle →

A reversible refrigeration cycle which would be a measure of max. possible COP of refrigerating machine operating between two temp. T_2 of heat rejection and T_0 of refrigeration.



A reversed Carnot Cycle for a unit mass of the working substance on T-S diagram.

Process 1-2 Isentropic compression $s_1 = s_2$

Process 2-3 Isothermal heat rejection to the hot reservoir at T_2 = constant.

Process 3-4 Isentropic expansion $s_3 = s_4$

Process 4-1 Isothermal heat absorption from the cold reservoir at T_0 = constant

Heat absorbed from cold body $Q_0 = T_0 \Delta S$

Heat rejected to hot body $Q_R = T_2 \Delta S$

Workdone $W = Q_R - Q_0 = (T_2 - T_0) \Delta S$

$$(COP)_{R\ carnot} = \frac{\dot{Q}_0}{W} = \frac{T_0}{T_2 - T_0}$$

$$(COP)_{HP\ carnot} = \frac{Q_2}{W} = \frac{T_2}{T_2 - T_0}$$

Note Carnot COP depends on the operating temp. T_2 & T_0. It does not depend on the working substance (refrigerant) used.

* The lowest possible refrigeration temp $T_0 = 0K$ (absolute zero) at which $(COP)_R = 0$. The highest possible refrigeration temp. $T_0 = T_2$ $(COP)_R = \infty$. Thus Carnot COP for cooling varies between 0 to ∞.

* For heating, min. $T_0 = 0K$ $(COP)_{HP} = 1$
max. $T_0 = T_2$ $(COP)_{HP} = \infty$ ie. $(COP)_{Heating}$ varies between 1 to ∞.

for eg. $T_0 = -25°C$ (248 K) $T_2 = 60°C$ (333 K)

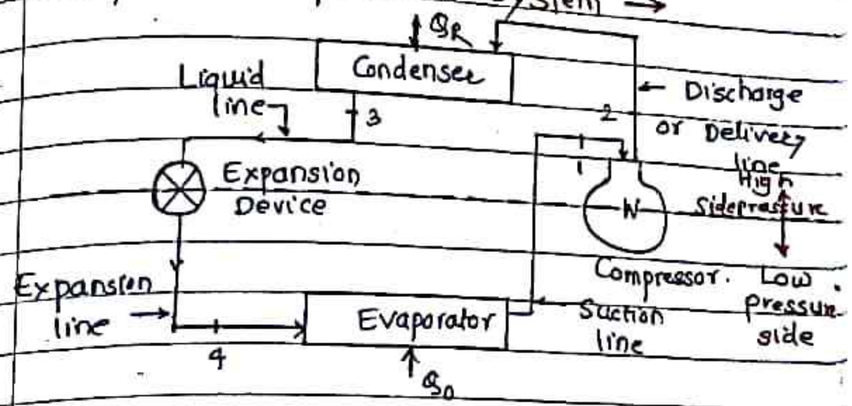
$$\text{Max. } (COP)_R = \frac{248}{333-248} = 2.9$$

$T_0 = 5°C$ (278 K) $T_2 = 60°C$ (333 K)

$$\text{Max. } (COP)_R = \frac{278}{333-278} = 5$$

Note COP of a refrigeration system decreases and power consumption increases as we go to lower & lower refrigeration temperatures.

Vapour Compression System →



It consists of a compressor, a condenser, an expansion device for throttling and an evaporator. The compressor - delivery head, discharge line, condenser and liquid line form the high pressure side of the system. The compressor and matching condenser together are also available commercially as one unit called the condensing unit. The expansion line evaporator, suction line & compressor - suction head form the low pressure side of the system. The expansion device is located as close to the evaporator as possible in order to minimize the heat gain in the low temp. expansion line.

In plants with a large amount of refrigerant charge, a receiver is installed in the liquid line. Normally a drier is also called installed in the liquid line particularly in flurocarbon systems. The drier contains silica gel and absorbs traces of moisture present in the liquid refrigerant so that it does not enter the narrow c/s of the expansion device causing moisture chocking by freezing.

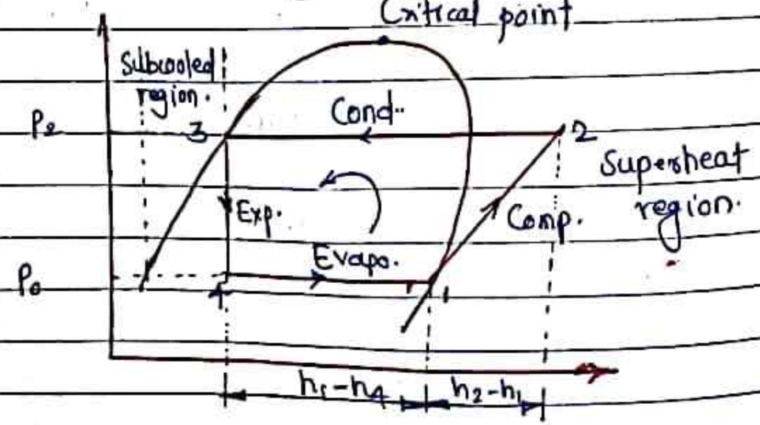
1) Process 1-2 Isentropic Compression $s_1 = s_2$
$Q = 0$

Workdone $W = -\int v\,dp = -\int dh = -(h_2 - h_1)$

2) Process 2-3 - Desuperheating & Condensation P_2 = Constant.
Heat rejected at $P_2 = q_2 = h_2 - h_3$

3) Process 3-4 - Isenthalpic expansion $h_3 = h_4$
$h_3 - h_4 = hf_4 + x_4(h_1 - hf_4)$
$x = \dfrac{h_3 - hf_4}{h_1 - hf_4}$

4) Process 4-1 Evaporation P_0 = constant
Refrigerating effect $= q_0 = h_1 - h_4$



Two constant pressure & one isenthalpic process hence it is convenient to draw p-h diagram & also workdone is given by the increase in enthalpy.

$P_c \rightarrow$ Saturated discharge pressures.
$P_0 \rightarrow$ Sat. suction pressure.

$(COP)_{cooling} = \dfrac{h_1 - h_4}{h_2 - h_1}$

$(COP)_{heating} = \dfrac{h_2 - h_3}{h_2 - h_1}$

Refrigerant Circulation rate $= \dot{m} = \dfrac{\text{Refrigerating Capacity}}{\text{Refri effect per unit mass}}$

$\dot{m} = \dfrac{\dot{Q}_0}{q_0}$

Specific volume of the vapour at suction $= v_1$

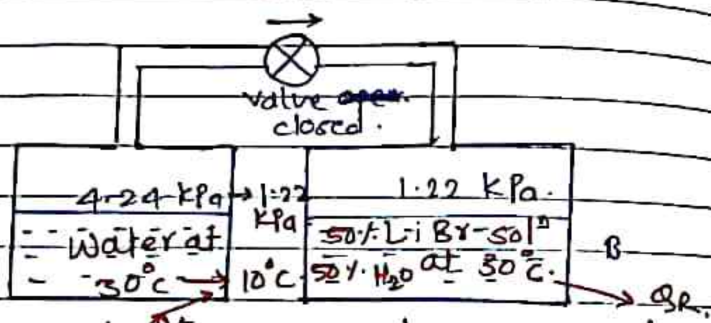
Theoretical piston displacement $\dot{V} = \dot{m} v_1$

* Vapour Absorption systems →

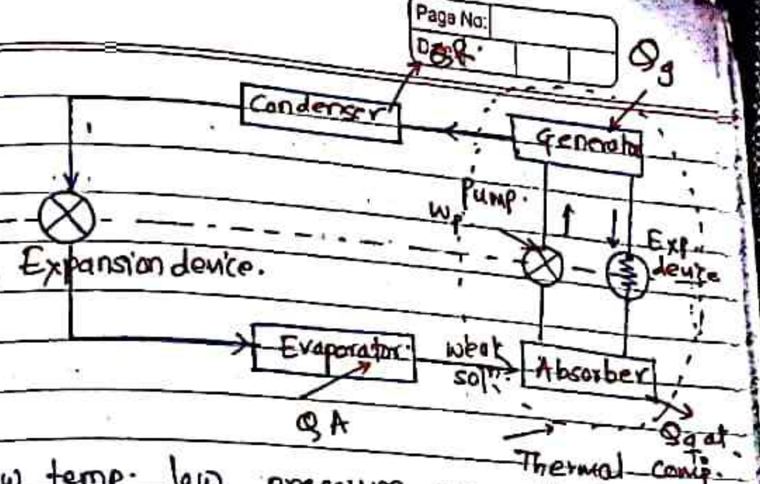
The required input to vapour absorption systems in the form of heat. Hence these systems are also called heat operated or thermal energy driven systems. Since these systems works on low grade energy hence widely used as waste heat or solar energy is available.

Solvent → Pure water

Solute → Lithium Bromide.



when valve opens water vapour from container A will flow to B due to pressure diff. & this vapour will absorbed by solution in B. Pressure in A reaches to equilibrium pressure 1.22 kPa & also reducing temp upto 10°c. Since water temp. is lower than the surrounding temp. & again heat absorbed by water, converts into vapour. This heat is absorbed by solution. This absorbed heat by solution is rejected to surrounding by some means.



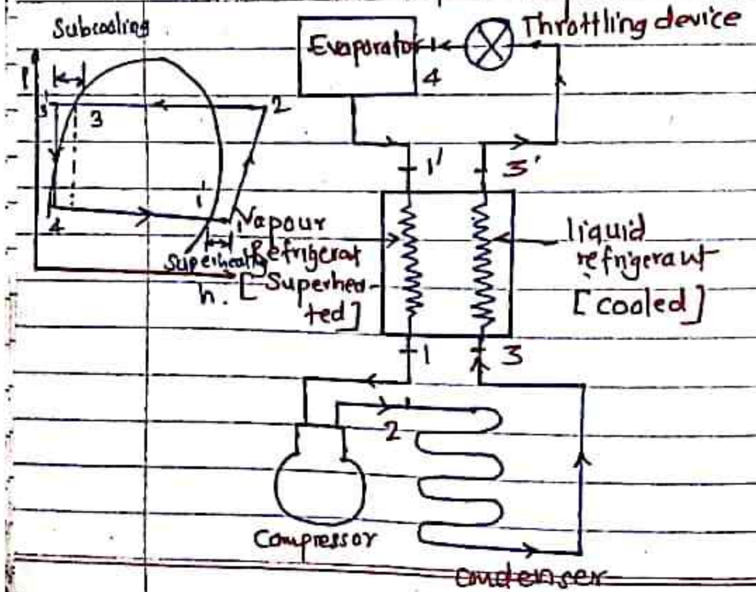
Low temp., low pressure ~~ve~~ with low quality enters a evaporator and vaporizes by producing useful refrigeration Q_A. From the evaporator, the low temp., low pressure refri- vapour enters in the absorber where it comes in contact with a solution that is weak in refrigerant. The weak solution absorbs the refrigerant & becomes strong in refrigerant. The heat of absorption is rejected to the external heat sink at T_0. The condensed liquid refrigerant is ~~they~~ with rich solution is fed to generated via pump. In the generator heat at high temp. T_g is provided as result refrigerant vapour is generated at high pressure. is fed to condenser while the hot, high pressure solution that is weak in refrigerant is throttled to absorber where it comes in contact with the refrigerant vapour from evaporator. Condenser reject the heat to surr. conv. liquid refrigerant. expanded into ~~tiff~~ expansion device.

The pump work $Q_p = -\int v\,dp$ is very small compared to compressor in VCRS as the specific volume v of liquid is extremely small compared to the vapour ($V_f <<< V_g$) ∴ The energy consumption of the system is mainly in the generator in the form of heat supplied Q_g.

$$COP = \frac{\text{Ref. effect}}{\text{Energy Supplied}}$$

$$\boxed{COP = \frac{Q_A}{Q_g + W_p}}$$

* VCRS with liquid vapour heat exchanger →



In liquid suction heat exchanger the cold suction vapour piped through heat exchanger to warm liquid refrigerant flowing through liquid line, to the expansion valve. While flowing through H.E. the cold suction vapour absorbs heat from liquid refrigerant so that liquid refrigerant is subcooled & temp. of cold suction vapour increases. Vapour becomes superheated vapour & liquid becomes subcooled liquid. Simultaneous subcooling & superheating of refrigerant increases COP of cycle.